International Baccalaureate®
Baccalauréat International
Bachillerato Internacional

Mathematics: applications and interpretation

Higher level and Standard level

Specimen papers 1, 2 and 3

First examinations in 2021

CONTENTS

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Higher level
Paper 1

Specimen paper

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

Please **do not** write on this page.

Answers written on this page
will not be marked.

Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 6]

At the end of a school day, the Headmaster conducted a survey asking students in how many classes they had used the internet.

The data is shown in the following table.

Number of classes in which the students used the internet	0	1	2	3	4	5	6
Number of students	20	24	30	k	10	3	1

(a) State whether the data is discrete or continuous. [1]

The mean number of classes in which a student used the internet is 2.

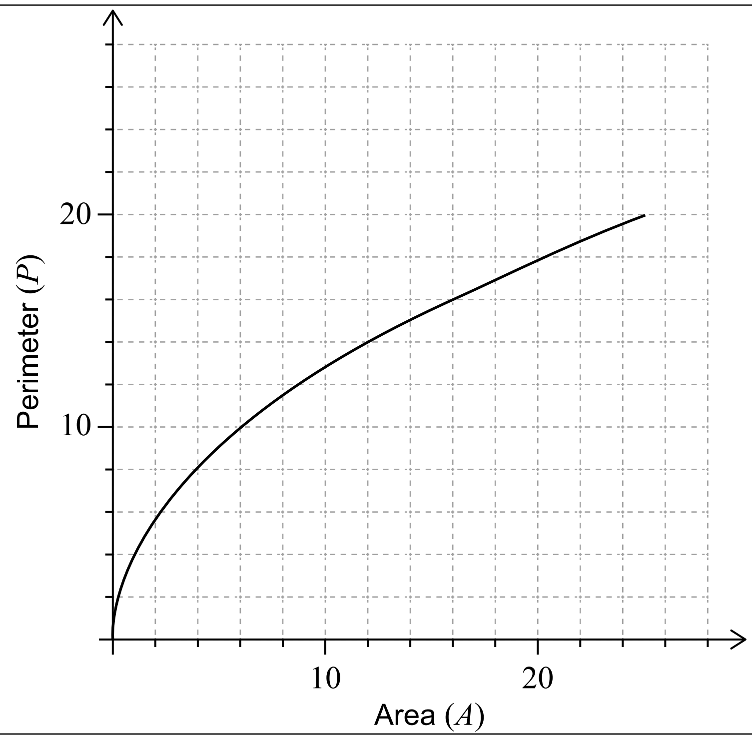
(b) Find the value of k. [4]

It was not possible to ask every person in the school, so the Headmaster arranged the student names in alphabetical order and then asked every 10th person on the list.

(c) Identify the sampling technique used in the survey. [1]

2. [Maximum mark: 5]

The perimeter of a given square P can be represented by the function $P(A) = 4\sqrt{A}$, $A \geq 0$, where A is the area of the square. The graph of the function P is shown for $0 \leq A \leq 25$.



(a) Write down the value of $P(25)$. [1]

(b) On the axes above, draw the graph of the inverse function, P^{-1}. [3]

(c) In the context of the question, explain the meaning of $P^{-1}(8) = 4$. [1]

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3. [Maximum mark: 6]

Professor Vinculum investigated the migration season of the Bulbul bird from their natural wetlands to a warmer climate.

He found that during the migration season their population, P could be modelled by $P = 1350 + 400(1.25)^{-t}$, $t \geq 0$, where t is the number of days since the start of the migration season.

(a) Find the population of the Bulbul birds,

(i) at the start of the migration season.

(ii) in the wetlands after 5 days. [3]

(b) Calculate the time taken for the population to decrease below 1400. [2]

(c) According to this model, find the smallest possible population of Bulbul birds during the migration season. [1]

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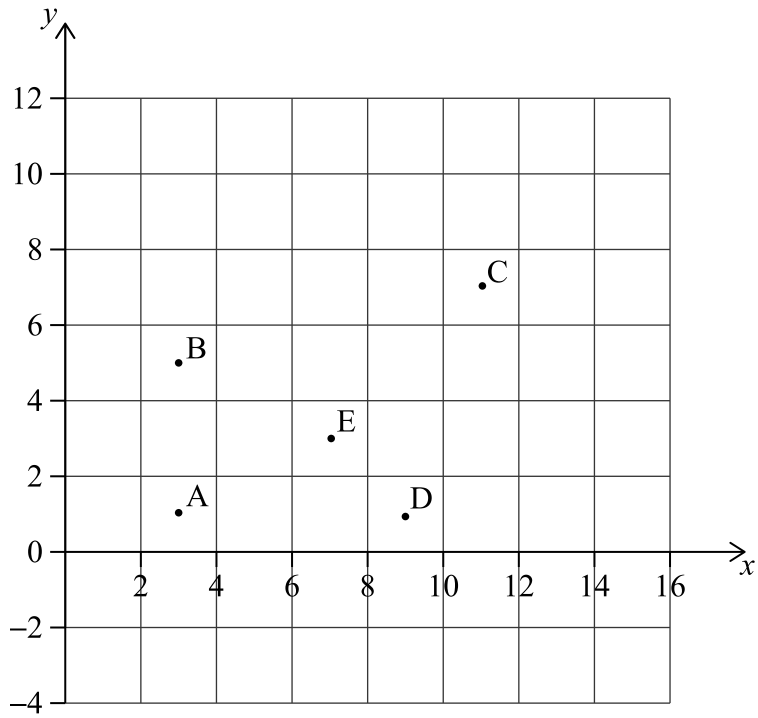
4. [Maximum mark: 6]

Points $A(3, 1)$, $B(3, 5)$, $C(11, 7)$, $D(9, 1)$ and $E(7, 3)$ represent snow shelters in the Blackburn National Forest. These snow shelters are illustrated in the following coordinate axes.

Horizontal scale: 1 unit represents $1\,\text{km}$.

Vertical scale: 1 unit represents $1\,\text{km}$.

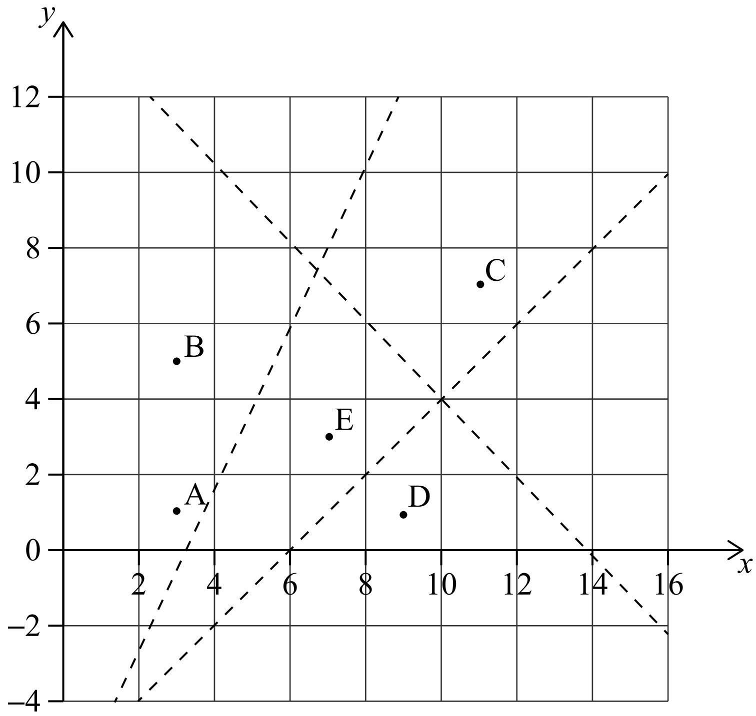


(a) Calculate the gradient of the line segment AE. [2]

(This question continues on the following page)

(Question 4 continued)

The Park Ranger draws three straight lines to form an incomplete Voronoi diagram.



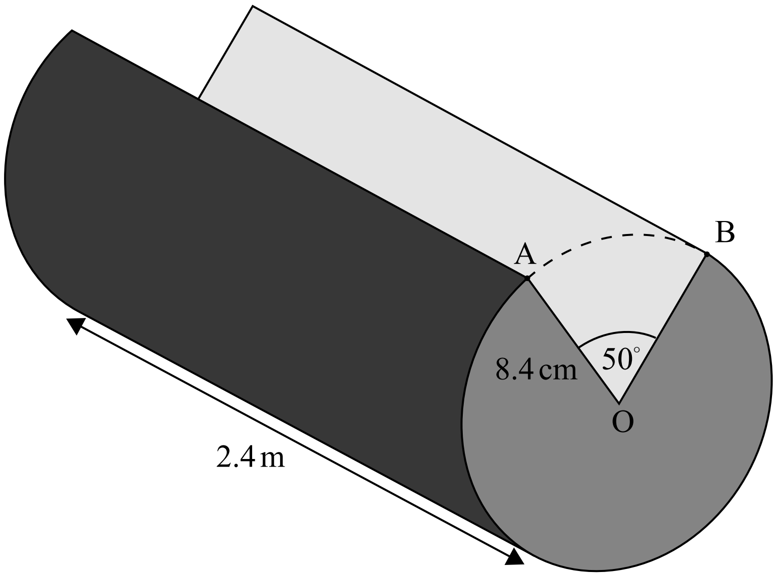
(b) Find the equation of the line which would complete the Voronoi cell containing site E. Give your answer in the form $ax + by + d = 0$ where $a, b, d \in \mathbb{Z}$. [3]

(c) In the context of the question, explain the significance of the Voronoi cell containing site E. [1]

5. [Maximum mark: 5]

Helen is building a cabin using cylindrical logs of length $2.4\,\text{m}$ and radius $8.4\,\text{cm}$. A wedge is cut from one log and the cross-section of this log is illustrated in the following diagram.

diagram not to scale



(a) Find 50° in radians. [1]

(b) Find the volume of this log. [4]

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6. [Maximum mark: 6]

Jae Hee plays a game involving a biased six-sided die.
The faces of the die are labelled -3, -1, 0, 1, 2 and 5.
The score for the game, X, is the number which lands face up after the die is rolled.
The following table shows the probability distribution for X.

Score x	-3	-1	0	1	2	5
$\mathbf{P}(X=x)$	$\frac{1}{18}$	p	$\frac{3}{18}$	$\frac{1}{18}$	$\frac{2}{18}$	$\frac{7}{18}$

(a) Find the exact value of p. [1]

Jae Hee plays the game once.

(b) Calculate the expected score. [2]

Jae Hee plays the game twice and adds the two scores together.

(c) Find the probability Jae Hee has a **total** score of -3. [3]

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7. [Maximum mark: 5]

A particle, A, moves so that its velocity ($v\,\text{m}\,\text{s}^{-1}$) at time t is given by $v = 2\sin t,\ t \geq 0$.

The kinetic energy (E) of the particle A is measured in joules (J) and is given by $E = 5v^2$.

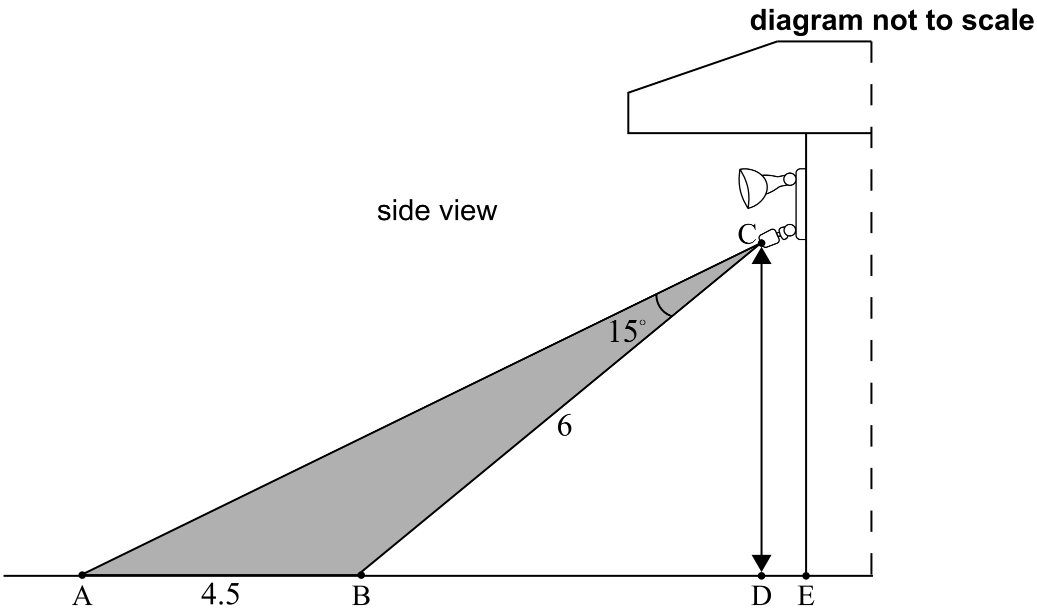
(a) Write down an expression for E as a function of time. [1]

(b) Hence find $\dfrac{\mathrm{d}E}{\mathrm{d}t}$. [2]

(c) Hence or otherwise find the first time at which the kinetic energy is changing at a rate of $5\,\text{J}\,\text{s}^{-1}$. [2]

8. [Maximum mark: 8]

Ollie has installed security lights on the side of his house that are activated by a sensor. The sensor is located at point C directly above point D. The area covered by the sensor is shown by the shaded region enclosed by triangle ABC. The distance from A to B is $4.5\,\text{m}$ and the distance from B to C is $6\,\text{m}$. Angle $\text{A}\hat{\text{C}}\text{B}$ is 15°.



(a) Find $\text{C}\hat{\text{A}}\text{B}$. [3]

Point B on the ground is $5\,\text{m}$ from point E at the entrance to Ollie's house. He is $1.8\,\text{m}$ tall and is standing at point D, below the sensor. He walks towards point B.

(b) Find the distance Ollie is **from the entrance to his house** when he first activates the sensor. [5]

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9. [Maximum mark: 5]

A manager wishes to check the mean weight of flour put into bags in his factory. He randomly samples 10 bags and finds the mean weight is $1.478\,\text{kg}$ and the standard deviation of the sample is $0.0196\,\text{kg}$.

(a) Find s_{n-1} for this sample. [2]

(b) Find a 95% confidence interval for the population mean, giving your answer to 4 significant figures. [2]

(c) The bags are labelled as being $1.5\,\text{kg}$ weight. Comment on this claim with reference to your answer in part (b). [1]

10. [Maximum mark: 6]

In a coffee shop, the time it takes to serve a customer can be modelled by a normal distribution with a mean of 1.5 minutes and a standard deviation of 0.4 minutes.

Two customers enter the shop together. They are served one at a time.

Find the probability that the total time taken to serve both customers will be less than 4 minutes.

Clearly state any assumptions you have made.

11. [Maximum mark: 6]

A particle P moves with velocity $\boldsymbol{v} = \begin{pmatrix} -15 \\ 2 \\ 4 \end{pmatrix}$ in a magnetic field, $\boldsymbol{B} = \begin{pmatrix} 0 \\ d \\ 1 \end{pmatrix}$, $d \in \mathbb{R}$.

(a) Given that $\boldsymbol{v}$ is perpendicular to $\boldsymbol{B}$, find the value of d. [2]

The force, $\boldsymbol{F}$, produced by P moving in the magnetic field is given by the vector equation $\boldsymbol{F} = a\boldsymbol{v} \times \boldsymbol{B}$, $a \in \mathbb{R}^{+}$.

(b) Given that $|\boldsymbol{F}| = 14$, find the value of a. [4]

12. [Maximum mark: 7]

Product research leads a company to believe that the revenue (R) made by selling its goods at a price (p) can be modelled by the equation.

$$R(p) = cp\mathrm{e}^{dp}, \ c, \ d \in \mathbb{R}$$

There are two competing models, A and B with different values for the parameters c and d.

Model A has $c = 3$, $d = -0.5$ and model B has $c = 2.5$, $d = -0.6$.

The company experiments by selling the goods at three different prices in three similar areas and the results are shown in the following table.

Area	**Price (p)**	**Revenue (R)**
1	1	1.5
2	2	1.8
3	3	1.5

The company will choose the model with the smallest value for the sum of square residuals.

Determine which model the company chose.

13. [Maximum mark: 6]

The rates of change of the area covered by two types of fungi, X and Y,on a particular tree are given by the following equations, where x is the area covered by X and y is the area covered by Y.

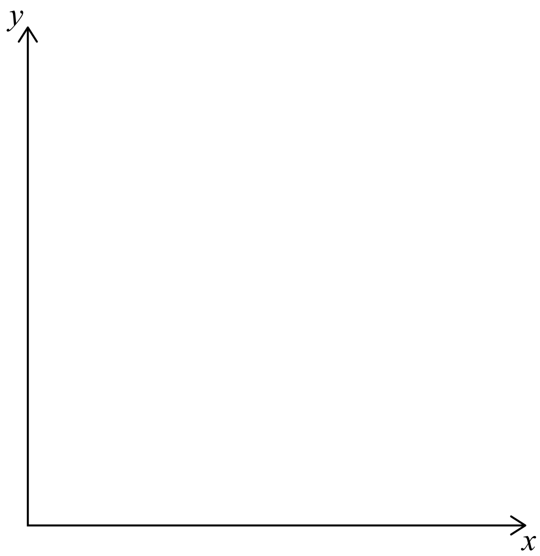
$$\frac{\mathrm{d}x}{\mathrm{d}t} = 3x - 2y$$
$$\frac{\mathrm{d}y}{\mathrm{d}t} = 2x - 2y$$

The matrix $\begin{pmatrix} 3 & -2 \\ 2 & -2 \end{pmatrix}$ has eigenvalues of 2 and -1 with corresponding eigenvectors $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$.

Initially $x = 8\,\mathrm{cm}^2$ and $y = 10\,\mathrm{cm}^2$.

(a) Find the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ when $t = 0$. [2]

(b) On the following axes, sketch a possible trajectory for the growth of the two fungi, making clear any asymptotic behaviour. [4]



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(Question 13 continued)

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Answers written on this page
will not be marked.

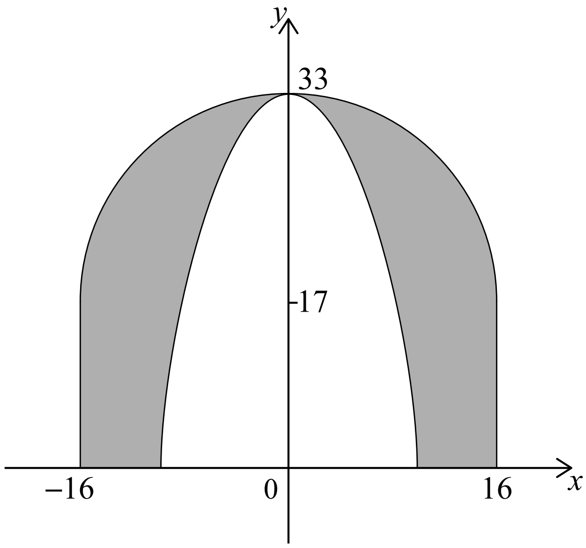
14. [Maximum mark: 8]

(a) The graph of $y = -x^3$ is transformed onto the graph of $y = 33 - 0.08x^3$ by a translation of a units vertically and a stretch parallel to the x-axis of scale factor b.

(i) Write down the value of a.

(ii) Find the value of b. [3]

(b) The outer dome of a large cathedral has the shape of a hemisphere of diameter $32\,\text{m}$, supported by vertical walls of height $17\,\text{m}$. It is also supported by an inner dome which can be modelled by rotating the curve $y = 33 - 0.08x^3$ through 360° about the y-axis between $y = 0$ and $y = 33$, as indicated in the diagram.



Find the volume of the space between the two domes. [5]

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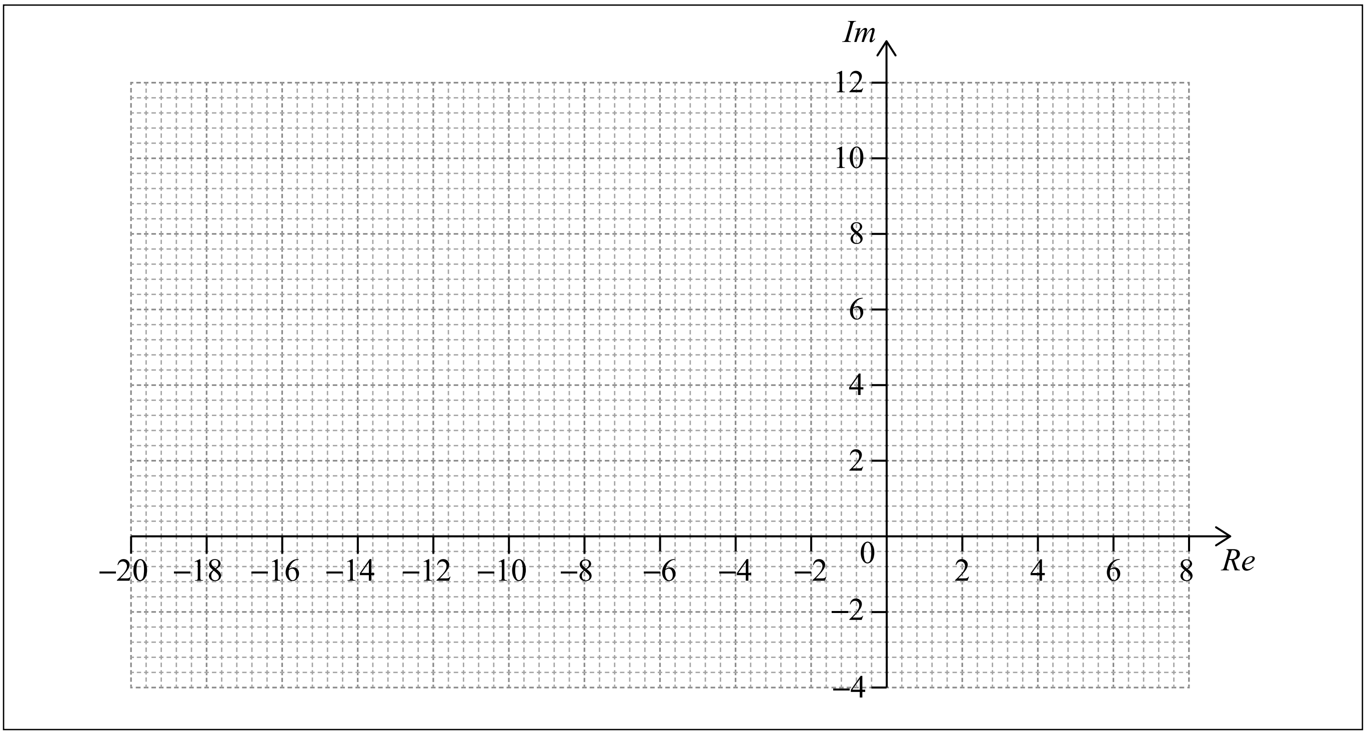
15. [Maximum mark: 7]

Let $w = a\mathrm{e}^{\frac{\pi}{4}\mathrm{i}}$, where $a \in \mathbb{R}^+$.

(a) For $a = 2$,

(i) find the values of w^2, w^3, and w^4;

(ii) draw w, w^2, w^3 and w^4 on the following Argand diagram. [5]



Let $z = \dfrac{w}{2 - \mathrm{i}}$.

(b) Find the value of a for which successive powers of z lie on a circle. [2]

(This question continues on the following page)

(Question 15 continued)

16. [Maximum mark: 6]

The number of fish that can be caught in one hour from a particular lake can be modelled by a Poisson distribution.

The owner of the lake, Emily, states in her advertising that the average number of fish caught in an hour is three.

Tom, a keen fisherman, is not convinced and thinks it is less than three. He decides to set up the following test. Tom will fish for one hour and if he catches fewer than two fish he will reject Emily's claim.

(a) State a suitable null and alternative hypotheses for Tom's test. [1]

(b) Find the probability of a Type I error. [2]

The average number of fish caught in an hour is actually 2.5.

(c) Find the probability of a Type II error. [3]

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17. [Maximum mark: 6]

Mr Burke teaches a mathematics class with 15 students. In this class there are 6 female students and 9 male students.

Each day Mr Burke randomly chooses one student to answer a homework question.

In the first month, Mr Burke will teach his class 20 times.

(a) Find the probability he will choose a female student 8 times. [2]

The Head of Year, Mrs Smith, decides to select a student at random from the year group to read the notices in assembly. There are 80 students in total in the year group. Mrs Smith calculates the probability of picking a male student 8 times in the first 20 assemblies is 0.153357 correct to 6 decimal places.

(b) Find the number of male students in the year group. [4]

18. [Maximum mark: 6]

The rate, A, of a chemical reaction at a fixed temperature is related to the concentration of two compounds, B and C, by the equation

$$A = kB^x C^y \text{, where } x,\ y,\ k \in \mathbb{R}.$$

A scientist measures the three variables three times during the reaction and obtains the following values.

Experiment	A **(mol l^{-1} s^{-1})**	B **(mol l^{-1})**	C **(mol l^{-1})**
1	5.74	2.1	3.4
2	2.88	1.5	2.4
3	0.980	0.8	1.9

Find x, y and k.

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Markscheme

Specimen paper

Mathematics: applications and interpretation

Higher level

Paper 1

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme eg ***M1***, ***A2***.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***M2***, ***A3***, *etc*., do **not** split the marks, unless there is a note.
- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award the final ***A1***. An exception to this may be in numerical answers, where a correct exact value is followed by an incorrect decimal. However, if the incorrect decimal is carried through to a subsequent part, and correct ***FT*** working shown, award ***FT*** marks as appropriate but do not award the final ***A1*** in that part.

  Examples

	Correct answer seen	**Further working seen**	**Action**
1.	$8\sqrt{2}$	$5.65685...$ *(incorrect decimal value)*	Award the final ***A1*** *(ignore the further working)*
2.	$\frac{1}{4}\sin 4x$	$\sin x$	Do not award the final ***A1***
3.	$\log a - \log b$	$\log(a-b)$	Do not award the final ***A1***

3 Implied marks

*Implied marks appear in **brackets e.g. (M1)**, and can only be awarded if **correct** work is seen or if implied in subsequent working.*

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

4 Follow through marks (only applied after an error is made)

*Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the answer (i.e. there is no working expected), then **FT** marks should be awarded if appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- Exceptions to this rule will be explicitly noted on the markscheme.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.

5 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). Apply a **MR** penalty of 1 mark to that question*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates’ own work does **not** constitute a misread, it is an error.
- The ***MR*** penalty can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme

- Alternative methods for complete questions are indicated by ***METHOD 1***, ***METHOD 2***, *etc*.
- Alternative solutions for part-questions are indicated by ***EITHER*** . . . ***OR***.

7 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

8 Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. There are two types of accuracy errors, and the final answer mark should not be awarded if these errors occur.

- **Rounding errors**: only applies to final answers not to intermediate steps.
- **Level of accuracy**: when this is not specified in the question the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures*.

9 Calculators

A GDC is required for this examination, but calculators with symbolic manipulation features/ CAS functionality are not allowed.

Calculator notation

The subject guide says:

Students must always use correct mathematical notation, not calculator notation.

Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

1. (a) discrete *A1*

[1 mark]

(b) $\dfrac{24+60+3k+40+15+6}{88+k}=2$ *M1A1*

> **Note:** Award ***M1*** for substitution into the formula for the mean, award ***A1*** for a correct equation.

attempt to solve their equation ***(M1)***

$k=31$ *A1*

[4 marks]

(c) systematic *A1*

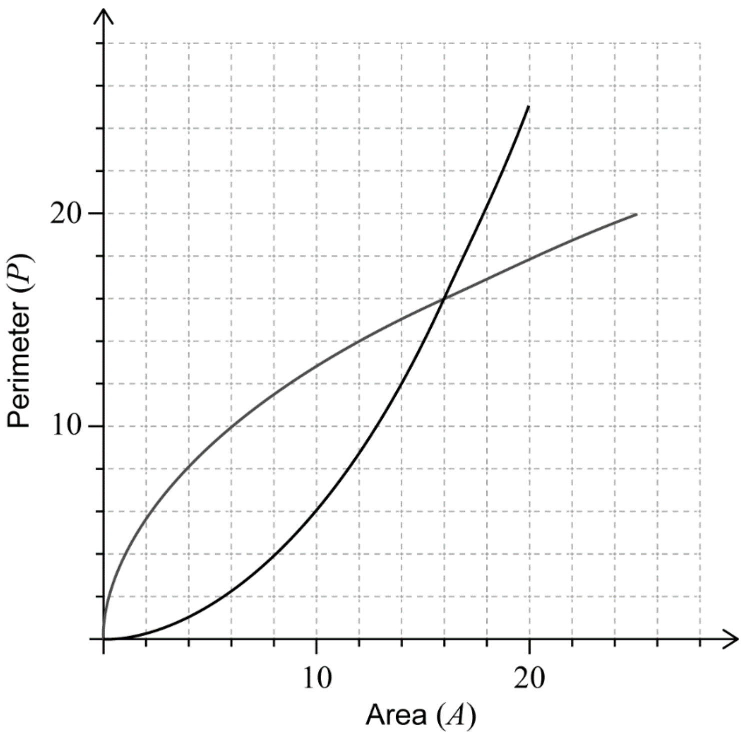
[1 mark]

Total [6 marks]

2. (a) 20 *A1*

[1 mark]

(b)



(M1)A1A1

> **Note:** Award ***(M1)*** for reflection in the line $P=A$, award ***A1*** for endpoint at $(20, 25)$, award ***A1*** for passing through $(16, 16)$.

[3 marks]

(c) when the perimeter is 8, the area is 4 *A1*

[1 mark]

Total [5 marks]

3. (a) (i) 1750 ***A1***

(ii) $1350 + 400\,(1.25)^{-5}$ ***(M1)***

$= 1480$ ***A1***

Note: Accept 1481.

[3 marks]

(b) $1400 = 1350 + 400\,(1.25)^{-t}$ ***(M1)***

9.32 (days ($9.31885\ldots$) (days)) ***A1***

[2 marks]

(c) 1350 ***A1***

Note: Accept 1351 as a valid interpretation of the model as $P = 1350$ is an asymptote.

[1 mark]

Total [6 marks]

4. (a) $\dfrac{3-1}{7-3}$ ***(M1)***

$= 0.5$ ***A1***

[2 marks]

(b) $y - 2 = -2(x - 5)$ ***(A1)(M1)***

Note: Award ***(A1)*** for their -2 seen, award ***(M1)*** for the correct substitution of $(5, 2)$ and their normal gradient in equation of a line.

$2x + y - 12 = 0$ ***A1***

[3 marks]

(c) every point in the cell is closer to E than any other snow shelter ***A1***

[1 mark]

Total [6 marks]

5. (a) $\frac{50 \times \pi}{180} = 0.873\ (0.872664\ldots)$ ***A1***

[1 mark]

(b) volume $= 240\left(\pi \times 8.4^2 - \frac{1}{2} \times 8.4^2 \times 0.872664\ldots\right)$ ***M1M1M1***

> **Note:** Award ***M1*** $240 \times$ area, award ***M1*** for correctly substituting area sector formula, award ***M1*** for subtraction of the angles or their areas.

$= 45800\ (= 45811.96071)$ ***A1***

[4 marks]

Total [5 marks]

6. (a) $\frac{4}{18}\left(\frac{2}{9}\right)$ ***A1***

[1 mark]

(b) $-3 \times \frac{1}{18} + (-1) \times \frac{4}{18} + 0 \times \frac{3}{18} + \ldots + 5 \times \frac{7}{18}$ ***(M1)***

> **Note:** Award ***(M1)*** for their correct substitution into the formula for expected value.

$= 1.83\left(\frac{33}{18}, 1.83333\ldots\right)$ ***A1***

[2 marks]

(c) $2 \times \frac{1}{18} \times \frac{3}{18}$ ***(M1)(M1)***

> **Note:** Award ***(M1)*** for $\frac{1}{18} \times \frac{3}{18}$, award ***(M1)*** for multiplying their product by 2.

$= \frac{1}{54}\left(\frac{6}{324}, 0.0185185\ldots, 1.85\%\right)$ ***A1***

[3 marks]

Total [6 marks]

7. (a) $E = 5(2\sin t)^2 \left(= 20\sin^2 t\right)$ ***A1***

[1 mark]

(b) $\dfrac{\mathrm{d}E}{\mathrm{d}t} = 40\sin t\cos t$ ***(M1)A1***

[2 marks]

(c) $t = 0.126$ ***(M1)A1***

[2 marks]

Total [5 marks]

8. (a) $\dfrac{\sin \mathrm{C\hat{A}B}}{6} = \dfrac{\sin 15^\circ}{4.5}$ ***(M1)(A1)***

$\mathrm{C\hat{A}B} = 20.2^\circ$ $(20.187415\ldots)$ ***A1***

Note: Award ***(M1)*** for substituted sine rule formula and award ***(A1)*** for correct substitutions.

[3 marks]

(b) $\mathrm{C\hat{B}D} = 20.2 + 15 = 35.2^\circ$ ***A1***

(let X be the point on BD where Ollie activates the sensor)

$\tan 35.18741\ldots^\circ = \dfrac{1.8}{\mathrm{BX}}$ ***(M1)***

Note: Award ***A1*** for their correct angle $\mathrm{C\hat{B}D}$. Award ***M1*** for correctly substituted trigonometric formula.

$\mathrm{BX} = 2.55285\ldots$ ***A1***

$5 - 2.55285\ldots$ ***(M1)***

$= 2.45\,(\mathrm{m})$ $(2.44714\ldots)$ ***A1***

[5 marks]

Total [8 marks]

9. (a) $s_{n-1} = \sqrt{\frac{10}{9}} \times 0.0196 = 0.02066\ldots$ ***(M1)A1***

[2 marks]

(b) $(1.463\,,\ 1.493)$ ***(M1)A1***

Note: If s_n used answer is $(1.464\,,\ 1.492)$, award ***M1A0.***

[2 marks]

(c) $95\,\%$ of the time these results would be produced by a population with mean of less than $1.5\,\text{kg}$, so it is likely the mean weight is less than $1.5\,\text{kg}$ ***R1***

[1 mark]

Total [5 marks]

10. let T be the time to serve both customers and T_i the time to serve the $i\,\text{th}$ customer

assuming independence of T_1 and T_2 ***R1***

T is normally distributed and $T = T_1 + T_2$ ***(M1)***

$E(T) = 1.5 + 1.5 = 3$ ***A1***

$\text{Var}(T) = 0.4^2 + 0.4^2 = 0.32$ ***M1A1***

$P(T < 4) = 0.961$ ***A1***

Total [6 marks]

11. (a) $15 \times 0 + 2d + 4 = 0$ ***(M1)***

$d = -2$ ***A1***

[2 marks]

(b) $a\begin{pmatrix} -15 \\ 2 \\ 4 \end{pmatrix} \times \begin{pmatrix} 0 \\ -2 \\ 1 \end{pmatrix}$ ***(M1)***

$= a\begin{pmatrix} 10 \\ 15 \\ 30 \end{pmatrix} \left(= 5a\begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix}\right)$ ***A1***

magnitude is $5a\sqrt{2^2 + 3^2 + 6^2} = 14$ ***M1***

$a = \dfrac{14}{35}$ $(= 0.4)$ ***A1***

[4 marks]

Total [6 marks]

12. *(Model A)*

$R = 3p\mathrm{e}^{-0.5p}$ ***M1***

predicted values

p	R
1	1.8196
2	2.2073
3	2.0082

(A1)

$SS_{res} = (1.8196 - 1.5)^2 + (2.2073 - 1.8)^2 + (2.0082 - 1.5)^2$ ***(M1)***

$= 0.5263\ldots$ ***A1***

(Model B)

$R = 2.5p\mathrm{e}^{-0.6p}$

predicted values

p	R
1	1.372
2	1.506
3	1.2397

(A1)

$SS_{res} = 0.170576\ldots$ ***A1***

chose model B ***A1***

Note: Method marks can be awarded if seen for either model A or model B. Award final ***A1*** if it is a correct deduction from their calculated values for A and B.

Total [7 marks]

13. (a) $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{16-20}{24-20}$ ***M1***

$= -1$ ***A1***

[2 marks]

(b) asymptote of trajectory along $\boldsymbol{r} = k\begin{pmatrix}2\\1\end{pmatrix}$ ***M1A1***

Note: Award ***M1A0*** if asymptote along $\begin{pmatrix}1\\2\end{pmatrix}$.

trajectory begins at $(8, 10)$ with negative gradient ***A1A1***

[4 marks]

Total [6 marks]

14. (a) (i) $a = 33$ ***A1***

(ii) $\frac{1}{\sqrt[3]{0.08}} = 2.32$ ***M1A1***

[3 marks]

(b) volume within outer dome

$\frac{2}{3}\pi \times 16^3 + \pi \times 16^2 \times 17 = 22\,250.85$ ***M1A1***

volume within inner dome

$\pi\int_0^{33}\left(\frac{33-y}{0.08}\right)^{\frac{2}{3}}\mathrm{d}y = 3446.92$ ***M1A1***

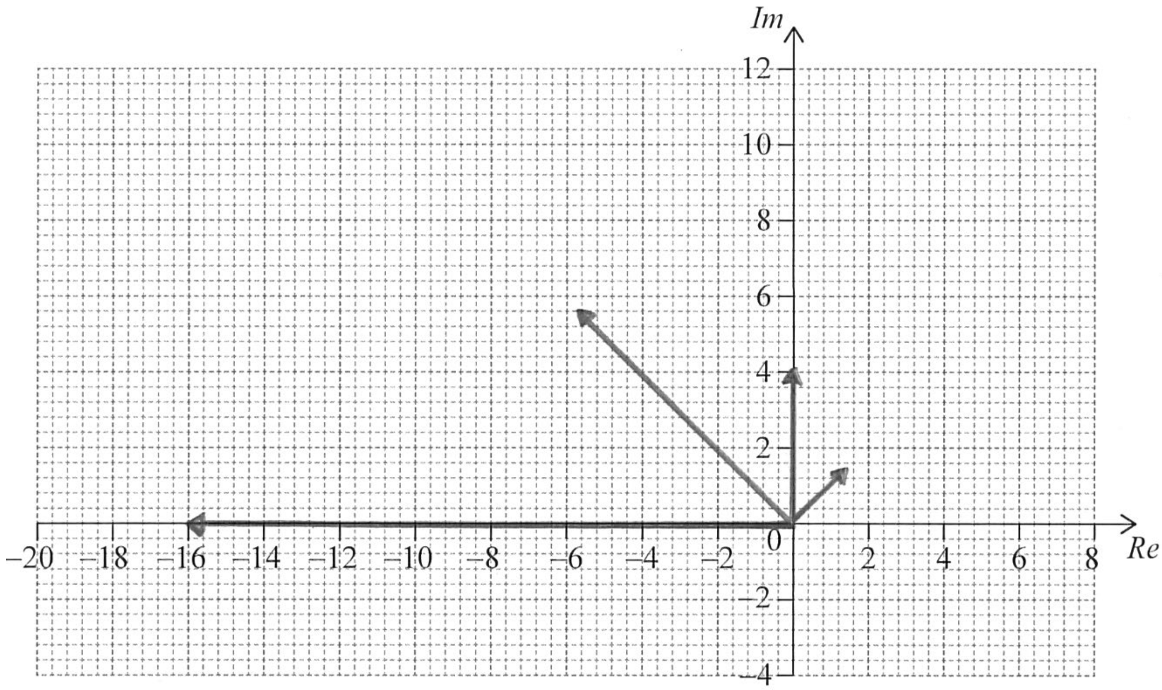
volume between $= 22\,250.85 - 3446.92 = 18\,803.93\,\mathrm{m}^3$ ***A1***

[5 marks]

Total [8 marks]

15. (a) (i) $4e^{\frac{\pi}{2}i}, 8e^{\frac{3\pi}{4}i}, 16e^{\pi i}$ $\left(= 4i, -4\sqrt{2} + 4\sqrt{2}i, -16\right)$ ***(M1)A1***

(ii)



A3

Note:	Award ***A1*** for correct arguments, award ***A1*** for $4i$ and -16 clearly indicated, award ***A1*** for $\lvert w \rvert < 4$ and $4 < \lvert w^3 \rvert < 16$.

[5 marks]

(b) $2^2 + 1^2 = a^2$ ***M1***

$a = \sqrt{5}$ $(= 2.24)$ ***A1***

[2 marks]

Total [7 marks]

16. (a) $H_0: m = 3,\ H_1: m < 3$ ***A1***

Note: Accept equivalent statements in words.

[1 mark]

(b) *(let X be the number of fish caught)*
$P(X \le 1 \mid m = 3) = 0.199$ ***M1A1***

[2 marks]

(c) $P(X \ge 2 \mid m = 2.5)\ \ (= 1 - P(X \le 1 \mid m = 2.5))$ ***M1A1***

Note: Award ***M1*** for using $m = 2.5$ to evaluate a probability, award ***A1*** for also having $X \ge 2$.

$= 0.713$ ***A1***

[3 marks]

Total [6 marks]

17. (a) $P(X = 8)$ ***(M1)***

Note: Award ***(M1)*** for evidence of recognizing binomial probability. *eg*, $P(X = 8),\ X \sim B\left(20, \frac{6}{15}\right)$.

$= 0.180\ (0.179705\ldots)$ ***A1***

[2 marks]

(b) let x be the number of male students

recognize that probability of selecting a male is equal to $\frac{x}{80}$ ***(A1)***

$\left(\text{set up equation } {}^{20}C_8\left(\frac{x}{80}\right)^8\left(\frac{80-x}{80}\right)^{12} = \right)\ 0.153357$ ***(M1)***

number of male students $= 37$ ***(M1)A1***

Note: Award ***(M1)A0*** for 27.

[4 marks]

Total [6 marks]

18. $\log A = x \log B + y \log C + \log k$ ***(M1)***

$\log 5.74 = x \log 2.1 + y \log 3.4 + \log k$
$\log 2.88 = x \log 1.5 + y \log 2.4 + \log k$
$\log 0.980 = x \log 0.8 + y \log 1.9 + \log k$ ***M1A1***

Note: Allow any consistent base, allow numerical equivalents.

attempting to solve their system of equations ***(M1)***

$x = 1.53,\ y = 0.505$ ***A1***

$k = 0.997$ ***A1***

Total [6 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Higher level
Paper 2

Specimen paper

2 hours

Instructions to candidates

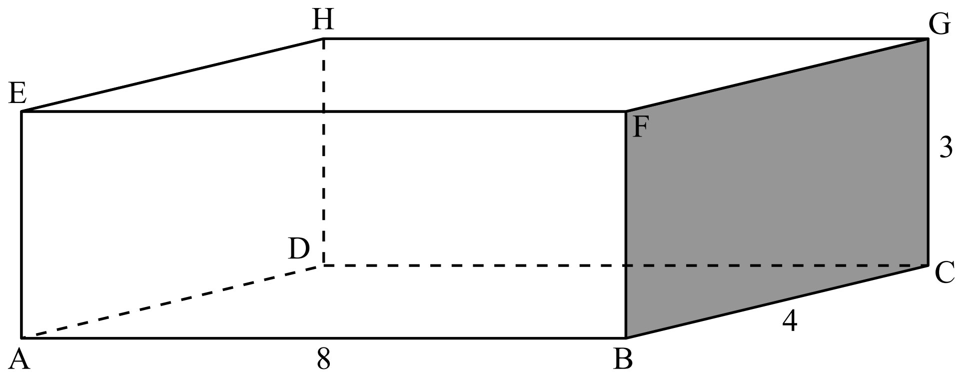
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

Answer **all** questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 15]

The Happy Straw Company manufactures drinking straws.

The straws are packaged in small closed rectangular boxes, each with length $8\,\text{cm}$, width $4\,\text{cm}$ and height $3\,\text{cm}$. The information is shown in the diagram.



(a) Calculate the surface area of the box in cm^2. [2]

(b) Calculate the length AG. [2]

Each week, the Happy Straw Company sells x boxes of straws. It is known that $\frac{dP}{dx} = -2x + 220$, $x \geq 0$, where P is the weekly profit, in dollars, from the sale of x thousand boxes.

(c) Find the number of boxes that should be sold each week to maximize the profit. [3]

The profit from the sale of $20\,000$ boxes is $\$1700$.

(d) Find $P(x)$. [5]

(e) Find the least number of boxes which must be sold each week in order to make a profit. [3]

2. [Maximum mark: 12]

Slugworth Candy Company sell a variety pack of colourful, shaped sweets.

The sweets are produced such that 80% are star shaped and 20% are shaped like a crescent moon. It is known that 10% of the stars and 30% of the crescent moons are coloured yellow.

(a) A sweet is selected at random.

(i) Find the probability that the sweet is yellow.

(ii) Given that the sweet is yellow, find the probability it is star shaped. [4]

According to manufacturer specifications, the colours in each variety pack should be distributed as follows.

Colour	Brown	Red	Green	Orange	Yellow	Purple
Percentage (%)	15	25	20	20	10	10

Mr Slugworth opens a pack of 80 sweets and records the frequency of each colour.

Colour	Brown	Red	Green	Orange	Yellow	Purple
Observed Frequency	10	20	16	18	12	4

To investigate if the sample is consistent with manufacturer specifications, Mr Slugworth conducts a χ^2 goodness of fit test. The test is carried out at a 5% significance level.

(b) Write down the null hypothesis for this test. [1]

(c) **Copy** and complete the following table in your answer booklet. [2]

Colour	Brown	Red	Green	Orange	Yellow	Purple
Expected Frequency						

(d) Write down the number of degrees of freedom. [1]

(e) Find the p-value for the test. [2]

(f) State the conclusion of the test. Give a reason for your answer. [2]

3. [Maximum mark: 18]

In this question, give all answers to two decimal places.

Bryan decides to purchase a new car with a price of €$14\,000$, but cannot afford the full amount. The car dealership offers two options to finance a loan.

Finance option A:

A 6 year loan at a nominal annual interest rate of 14% **compounded quarterly**. No deposit required and repayments are made each quarter.

(a) (i) Find the repayment made each quarter.

(ii) Find the total amount paid for the car.

(iii) Find the interest paid on the loan. [7]

Finance option B:

A 6 year loan at a nominal annual interest rate of $r\%$ **compounded monthly**. Terms of the loan require a 10% deposit and monthly repayments of €250.

(b) (i) Find the amount to be borrowed for this option.

(ii) Find the annual interest rate, r. [5]

(c) State which option Bryan should choose. Justify your answer. [2]

Bryan chooses option B. The car dealership invests the money Bryan pays as soon as they receive it.

(d) If they invest it in an account paying 0.4% interest per month and inflation is 0.1% per month, calculate the real amount of money the car dealership has received by the end of the 6 year period. [4]

4. [Maximum mark: 14]

An aircraft's position is given by the coordinates (x, y, z), where x and y are the aircraft's displacement east and north of an airport, and z is the height of the aircraft above the ground. All displacements are given in kilometres.

The velocity of the aircraft is given as $\begin{pmatrix} -150 \\ -50 \\ -20 \end{pmatrix} \mathrm{km\,h^{-1}}$.

At $13{:}00$ it is detected at a position $30\,\mathrm{km}$ east and $10\,\mathrm{km}$ north of the airport, and at a height of $5\,\mathrm{km}$. Let t be the length of time in hours from $13{:}00$.

(a) Write down a vector equation for the displacement, $\boldsymbol{r}$ of the aircraft in terms of t. [2]

(b) If the aircraft continued to fly with the velocity given

(i) verify that it would pass directly over the airport;

(ii) state the height of the aircraft at this point;

(iii) find the time at which it would fly directly over the airport. [4]

When the aircraft is $4\,\mathrm{km}$ above the ground it continues to fly on the same bearing but adjusts the angle of its descent so that it will land at the point $(0, 0, 0)$.

(c) (i) Find the time at which the aircraft is $4\,\mathrm{km}$ above the ground.

(ii) Find the direct distance of the aircraft from the airport at this point. [5]

(d) Given that the velocity of the aircraft, after the adjustment of the angle of descent, is $\begin{pmatrix} -150 \\ -50 \\ a \end{pmatrix} \mathrm{km\,h^{-1}}$, find the value of a. [3]

5. [Maximum mark: 17]

The following table shows the costs in US dollars (US$) of direct flights between six cities. Blank cells indicate no direct flights. The rows represent the departure cities. The columns represent the destination cities.

		Destination city					
		A	**B**	**C**	**D**	**E**	**F**
Departure city	**A**		90	150			
	B	90		80	70	140	
	C	150	80				
	D		70			100	180
	E		140		100		210
	F				180	210	

(a) Show the direct flights between the cities as a graph. [2]

(b) Write down the adjacency matrix for this graph. [2]

(c) Using your answer to part (b), find the number of different ways to travel from and return to city A in exactly 6 flights. [2]

(d) State whether or not it is possible to travel from and return to city A in exactly 6 flights, having visited each of the other 5 cities exactly once. Justify your answer. [2]

The following table shows the least cost to travel between the cities.

		Destination city					
		A	**B**	**C**	**D**	**E**	**F**
Departure city	**A**	0	90	150	160	a	b
	B	90	0	80	70	140	250
	C	150	80	0	150	220	330
	D	160	70	150	0	100	180
	E	a	140	220	100	0	210
	F	b	250	330	180	210	0

(e) Find the values of a and b. [2]

A travelling salesman has to visit each of the cities, starting and finishing at city A.

(f) Use the nearest neighbour algorithm to find an upper bound for the cost of the trip. [3]

(g) By deleting vertex A, use the deleted vertex algorithm to find a lower bound for the cost of the trip. [4]

6. [Maximum mark: 14]

A city has two cable companies, X and Y. Each year 20% of the customers using company X move to company Y and 10% of the customers using company Y move to company X. All additional losses and gains of customers by the companies may be ignored.

(a) Write down a transition matrix $\boldsymbol{T}$ representing the movements between the two companies in a particular year. [2]

(b) Find the eigenvalues and corresponding eigenvectors of $\boldsymbol{T}$. [4]

(c) Hence write down matrices $\boldsymbol{P}$ and $\boldsymbol{D}$ such that $\boldsymbol{T} = \boldsymbol{P}\boldsymbol{D}\boldsymbol{P}^{-1}$. [2]

Initially company X and company Y both have 1200 customers.

(d) Find an expression for the number of customers company X has after n years, where $n \in \mathbb{N}$. [5]

(e) Hence write down the number of customers that company X can expect to have in the long term. [1]

7. [Maximum mark: 20]

An object is placed into the top of a long vertical tube, filled with a thick viscous fluid, at time $t = 0$.

Initially it is thought that the resistance of the fluid would be proportional to the velocity of the object. The following model was proposed, where the object's displacement, x, from the top of the tube, measured in metres, is given by the differential equation

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = 9.81 - 0.9\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right).$$

(a) By substituting $v = \frac{\mathrm{d}x}{\mathrm{d}t}$ into the equation, find an expression for the velocity of the particle at time t. Give your answer in the form $v = f(t)$. [7]

The maximum velocity approached by the object as it falls is known as the terminal velocity.

(b) From your solution to part (a), or otherwise, find the terminal velocity of the object predicted by this model. [2]

An experiment is performed in which the object is placed in the fluid on a number of occasions and its terminal velocity recorded. It is found that the terminal velocity was consistently smaller than that predicted by the model used. It was suggested that the resistance to motion is actually proportional to the velocity squared and so the following model was set up.

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = 9.81 - 0.9\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2$$

(c) Write down the differential equation as a system of first order differential equations. [2]

(d) Use Euler's method, with a step length of 0.2, to find the displacement and velocity of the object when $t = 0.6$. [4]

(e) By repeated application of Euler's method, find an approximation for the terminal velocity, to five significant figures. [1]

At terminal velocity the acceleration of an object is equal to zero.

(f) Use the differential equation to find the terminal velocity for the object. [2]

(g) Use your answers to parts (d), (e) and (f) to comment on the accuracy of the Euler approximation to this model. [2]

Markscheme

Specimen paper

Mathematics: applications and interpretation

Higher level

Paper 2

14 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

*Award marks using the annotations as noted in the markscheme eg **M1**, **A2**.*

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***M2***, ***A3***, *etc*., do **not** split the marks, unless there is a note.
- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award the final ***A1***. An exception to this may be in numerical answers, where a correct exact value is followed by an incorrect decimal. However, if the incorrect decimal is carried through to a subsequent part, and correct ***FT*** working shown, award ***FT*** marks as appropriate but do not award the final ***A1*** in that part.

Examples

	Correct answer seen	**Further working seen**	**Action**
1.	$8\sqrt{2}$	$5.65685...$ *(incorrect decimal value)*	Award the final ***A1*** *(ignore the further working)*
2.	$\frac{1}{4}\sin 4x$	$\sin x$	Do not award the final ***A1***
3.	$\log a - \log b$	$\log(a-b)$	Do not award the final ***A1***

3 Implied marks

*Implied marks appear in **brackets e.g. (M1)**, and can only be awarded if **correct** work is seen or if implied in subsequent working.*

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

4 Follow through marks (only applied after an error is made)

*Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the answer (i.e. there is no working expected), then **FT** marks should be awarded if appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- Exceptions to this rule will be explicitly noted on the markscheme.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.

5 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). Apply a **MR** penalty of 1 mark to that question*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates’ own work does **not** constitute a misread, it is an error.
- The ***MR*** penalty can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme

- Alternative methods for complete questions are indicated by ***METHOD 1***, ***METHOD 2***, *etc*.
- Alternative solutions for part-questions are indicated by ***EITHER*** . . . ***OR***.

7 Alternative forms

Unless the question specifies otherwise, ***accept*** *equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

8 Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. There are two types of accuracy errors, and the final answer mark should not be awarded if these errors occur.

- **Rounding errors**: only applies to final answers not to intermediate steps.
- **Level of accuracy**: when this is not specified in the question the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures*.

9 Calculators

A GDC is required for this examination, but calculators with symbolic manipulation features/ CAS functionality are not allowed.

Calculator notation
The subject guide says:

Students must always use correct mathematical notation, not calculator notation.

Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

1. (a) $2(8\times4+3\times4+3\times8)$ ***M1***

$=136\ \ (\text{cm}^2)$ ***A1***

[2 marks]

(b) $\sqrt{8^2+4^2+3^2}$ ***M1***

$(\text{AG} =)\ 9.43\ (\text{cm})\ (9.4339\ldots,\ \sqrt{89})$ ***A1***

[2 marks]

(c) $-2x+220=0$ ***M1***

$x=110$ ***A1***

$110\,000$ (boxes) ***A1***

[3 marks]

(d) $P(x)=\int -2x+220\,\mathrm{d}x$ ***M1***

Note: Award ***M1*** for evidence of integration.

$P(x)=-x^2+220x+c$ ***A1A1***

Note: Award ***A1*** for either $-x^2$ or $220x$ award ***A1*** for both correct terms and constant of integration.

$1700=-(20)^2+220(20)+c$ ***M1***

$c=-2300$

$P(x)=-x^2+220x-2300$ ***A1***

[5 marks]

(e) $-x^2+220x-2300=0$ ***M1***

$x=11.005$ ***A1***

$11\,006$ (boxes) ***A1***

Note: Award ***M1*** for their $P(x)=0$, award ***A1*** for their correct solution to x. Award the final ***A1*** for expressing their solution to the minimum number of boxes. Do not accept $11\,005$, the nearest integer, nor $11\,000$, the answer expressed to 3 significant figures, as these will not satisfy the demand of the question.

[3 marks]

Total [15 marks]

2. (a) (i) $P(Y) = 0.8 \times 0.1 + 0.2 \times 0.3$ ***M1***

$= 0.14$ ***A1***

(ii) $P(\text{Star} \mid Y) = \dfrac{0.8 \times 0.1}{0.14}$ ***M1***

$= 0.571 \left(\dfrac{4}{7}, 0.571428\ldots \right)$ ***A1***

[4 marks]

(b) the colours of the sweets are distributed according to manufacturer specifications ***A1***

[1 mark]

(c)

Colour	Brown	Red	Green	Orange	Yellow	Purple
Expected Frequency	12	20	16	16	8	8

A2

Note: Award ***A2*** for all 6 correct expected values, ***A1*** for 4 or 5 correct values, ***A0*** otherwise.

[2 marks]

(d) 5 ***A1***

[1 mark]

(e) 0.469 (0.4688117…) ***A2***

[2 marks]

(f) since $0.469 > 0.05$ ***R1***

fail to reject the null hypothesis. There is insufficient evidence to reject the manufacturer's specifications ***A1***

Note: Award ***R1*** for a correct comparison of their correct p-value to the test level, award ***A1*** for the correct result from that comparison. Do not award ***R0A1***.

[2 marks]

Total [12 marks]

3. (a) (i) $\mathrm{N} = 24$

$\mathrm{I\%} = 14$

$\mathrm{PV} = -14\,000$

$\mathrm{FV} = 0$

$\mathrm{P/Y} = 4$

$\mathrm{C/Y} = 4$ ***(M1)(A1)***

> **Note:** Award ***M1*** for an attempt to use a financial app in their technology, award ***A1*** for all entries correct. Accept $\mathrm{PV} = 14\,000$.

$(€)871.82$ ***A1***

(ii) $4 \times 6 \times 871.82$ ***(M1)***

$(€)20\,923.68$ ***A1***

(iii) $20\,923.68 - 14\,000$ ***(M1)***

$(€)6923.68$ ***A1***

[7 marks]

(b) (i) $0.9 \times 14\,000\ (= 14\,000 - 0.10 \times 14\,000)$ ***M1***

$(€)12\,600.00$ ***A1***

(ii) $\mathrm{N} = 72$

$\mathrm{PV} = 12\,600$

$\mathrm{PMT} = -250$

$\mathrm{FV} = 0$

$\mathrm{P/Y} = 12$

$\mathrm{C/Y} = 12$ ***(M1)(A1)***

> **Note:** Award ***M1*** for an attempt to use a financial app in their technology, award ***A1*** for all entries correct. Accept $\mathrm{PV} = -12\,600$ provided $\mathrm{PMT} = 250$.

$12.56(\%)$ ***A1***

[5 marks]

continued…

Question 3 continued

(c) ***EITHER***

Bryan should choose Option A ***A1***

no deposit is required ***R1***

Note: Award ***R1*** for stating that no deposit is required. Award ***A1*** for the correct choice from that fact. Do not award ***R0A1.***

OR

Bryan should choose Option B ***A1***

cost of Option A $(6923.69) >$ cost of Option B $(72 \times 250 - 12\,600 = 5400)$ ***R1***

Note: Award ***R1*** for a correct comparison of costs. Award ***A1*** for the correct choice from that comparison. Do not award ***R0A1***.

[2 marks]

(d) real interest rate is $0.4 - 0.1 = 0.3\%$ ***(M1)***

value of other payments $250 + 250 \times 1.003 + \ldots + 250 \times 1.003^{71}$
use of sum of geometric sequence formula or financial app on a GDC ***(M1)***

$= 20\,058.43$

value of deposit at the end of 6 years
$1400 \times (1.003)^{72} = 1736.98$ ***(A1)***

Total value is $(€)\,21\,795.41$ ***A1***

Note: Both ***M*** marks can awarded for a correct use of the GDC's financial app:

$\text{N} = 72\ \ (6 \times 12)$
$\text{I\%} = 3.6\ \ (0.3 \times 12)$
$\text{PV} = 0$
$\text{PMT} = -250$
$\text{FV} =$
$\text{P/Y} = 12$
$\text{C/Y} = 12$

OR

$\text{N} = 72\ \ (6 \times 12)$
$\text{I\%} = 0.3$
$\text{PV} = 0$
$\text{PMT} = -250$
$\text{FV} =$
$\text{P/Y} = 1$
$\text{C/Y} = 1$

[4 marks]

Total [18 marks]

4. (a) $\boldsymbol{r} = \begin{pmatrix} 30 \\ 10 \\ 5 \end{pmatrix} + t\begin{pmatrix} -150 \\ -50 \\ -20 \end{pmatrix}$ ***A1A1***

[2 marks]

(b) (i) when $x = 0$, $t = \frac{30}{150} = 0.2$ ***M1***

EITHER

when $y = 0$, $t = \frac{10}{150} = 0.2$ ***A1***

since the two values of t are equal the aircraft passes directly over the airport

OR

$t = 0.2$, $y = 0$ ***A1***

(ii) height $= 5 - 0.2 \times 20 = 1\text{km}$ ***A1***

(iii) time $13{:}12$ ***A1***

[4 marks]

(c) (i) $5 - 20t = 4 \Rightarrow t = \frac{1}{20}$ (3 minutes) ***(M1)***

time $13{:}03$ ***A1***

(ii) displacement is $\begin{pmatrix} 22.5 \\ 7.5 \\ 4 \end{pmatrix}$ ***A1***

distance is $\sqrt{22.5^2 + 7.5^2 + 4^2}$ ***(M1)***

$= 24.1\text{km}$ ***A1***

[5 marks]

continued…

Question 4 continued

(d) ***METHOD 1***

time until landing is $12-3=9$ minutes **M1**

height to descend $=4\,\text{km}$

$a=\dfrac{-4}{\frac{9}{60}}$ **M1**

$=-26.7$ **A1**

METHOD 2

$\begin{pmatrix}-150\\-50\\a\end{pmatrix}=s\begin{pmatrix}22.5\\7.5\\4\end{pmatrix}$ **M1**

$-150=22.5s\Rightarrow s=-\dfrac{20}{3}$ **M1**

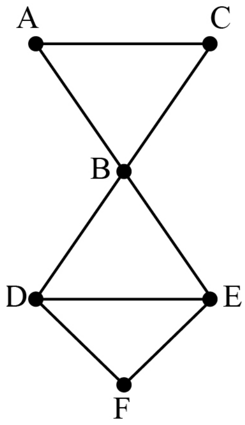
$a=-\dfrac{20}{3}\times 4$

$=-26.7$ **A1**

[3 marks]

Total [14 marks]

5. (a)



A2

[2 marks]

(b) attempt to form an adjacency matrix *M1*

$$\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$ *A1*

[2 marks]

(c) raising the matrix to the power six *(M1)*

50 *A1*

[2 marks]

(d) not possible *A1*

because you must pass through B twice *R1*

Note: Do not award ***A1R0.***

[2 marks]

(e) $a = 230$, $b = 340$ *A1A1*

[2 marks]

(f) $\mathrm{A} \rightarrow \mathrm{B} \rightarrow \mathrm{D} \rightarrow \mathrm{E} \rightarrow \mathrm{F} \rightarrow \mathrm{C} \rightarrow \mathrm{A}$ *(M1)*

$90 + 70 + 100 + 210 + 330 + 150$ *(A1)*

(US$) 950 *A1*

[3 marks]

continued…

Question 5 continued

(g) finding weight of minimum spanning tree **M1**

$70 + 80 + 100 + 180 =$ (US\$) 430 **A1**

adding in two edges of minimum weight **M1**

$430 + 90 + 150 =$ (US\$) 670 **A1**

[4 marks]

Total [17 marks]

6. (a) $\begin{pmatrix} 0.8 & 0.1 \\ 0.2 & 0.9 \end{pmatrix}$ **M1A1**

[2 marks]

(b) $\begin{vmatrix} 0.8-\lambda & 0.1 \\ 0.2 & 0.9-\lambda \end{vmatrix} = 0$ **M1**

$\lambda = 1$ and 0.7 **A1**

eigenvectors $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ **(M1)A1**

Note: Accept any scalar multiple of the eigenvectors.

[4 marks]

(c) ***EITHER***

$\boldsymbol{P} = \begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}$ $\boldsymbol{D} = \begin{pmatrix} 1 & 0 \\ 0 & 0.7 \end{pmatrix}$ **A1A1**

OR

$\boldsymbol{P} = \begin{pmatrix} 1 & 1 \\ -1 & 2 \end{pmatrix}$ $\boldsymbol{D} = \begin{pmatrix} 0.7 & 0 \\ 0 & 1 \end{pmatrix}$ **A1A1**

[2 marks]

(d) $\boldsymbol{P}^{-1} = \frac{1}{3}\begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}$ **A1**

$\frac{1}{3}\begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 0.7^n \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} 1200 \\ 1200 \end{pmatrix}$ **M1A1**

attempt to multiply matrices **M1**

so in company A, after n years, $400\left(2 + 0.7^n\right)$ **A1**

[5 marks]

(e) $400 \times 2 = 800$ **A1**

[1 mark]

Total [14 marks]

7. (a) $\frac{\mathrm{d}v}{\mathrm{d}t} = 9.81 - 0.9v$ ***M1***

$\int \frac{1}{9.81 - 0.9v}\,\mathrm{d}v = \int 1\,\mathrm{d}t$ ***M1***

$-\frac{1}{0.9}\ln(9.81 - 0.9v) = t + c$ ***A1***

$9.81 - 0.9v = A\mathrm{e}^{-0.9t}$ ***A1***

$v = \frac{9.81 - A\mathrm{e}^{-0.9t}}{0.9}$ ***A1***

when $t = 0$, $v = 0$ hence $A = 9.81$ ***A1***

$v = \frac{9.81(1 - \mathrm{e}^{-0.9t})}{0.9}$

$v = 10.9(1 - \mathrm{e}^{-0.9t})$ ***A1***

[7 marks]

(b) **either** let t tend to infinity, or $\frac{\mathrm{d}v}{\mathrm{d}t} = 0$ ***(M1)***

$v = 10.9$ ***A1***

[2 marks]

(c) $\frac{\mathrm{d}x}{\mathrm{d}t} = y$ ***M1***

$\frac{\mathrm{d}y}{\mathrm{d}t} = 9.81 - 0.9y^2$ ***A1***

[2 marks]

(d) $x_{n+1} = x_n + 0.2y_n$, $y_{n+1} = y_n + 0.2(9.81 - 0.9(y_n)^2)$ ***(M1)(A1)***

$x = 1.04, \frac{\mathrm{d}x}{\mathrm{d}t} = 3.31$ ***(M1)A1***

[4 marks]

(e) 3.3015 ***A1***

[1 mark]

(f) $0 = 9.81 - 0.9(v)^2$ ***M1***

$\Rightarrow v = \sqrt{\frac{9.81}{0.9}} = 3.301511\ldots\ (= 3.30)$ ***A1***

[2 marks]

continued…

Question 7 continued

(g) the model found the terminal velocity very accurately, so good approximation ***R1***

intermediate values had object exceeding terminal velocity so not good approximation ***R1***

[2 marks]

Total [20 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Higher level
Paper 3

Specimen paper

1 hour

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[55 marks]**.

Answer **both** questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 27]

Two IB schools, A and B, follow the IB Diploma Programme but have different teaching methods. A research group tested whether the different teaching methods lead to a similar final result.

For the test, a group of eight students were randomly selected from each school. Both samples were given a standardized test at the start of the course and a prediction for total IB points was made based on that test; this was then compared to their points total at the end of the course.

Previous results indicate that both the predictions from the standardized tests and the final IB points can be modelled by a normal distribution.

It can be assumed that:

- the standardized test is a valid method for predicting the final IB points
- that variations from the prediction can be explained through the circumstances of the student or school.

(a) Identify a test that might have been used to verify the null hypothesis that the predictions from the standardized test can be modelled by a normal distribution. [1]

(b) State why comparing only the final IB points of the students from the two schools would not be a valid test for the effectiveness of the two different teaching methods. [1]

(This question continues on the following page)

(Question 1 continued)

The data for school A is shown in the following table.

School A

Student number	Gender	Predicted IB points (p)	Final IB points (f)
1	male	43.2	44
2	male	36.5	34
3	female	37.1	38
4	male	30.9	28
5	male	41.1	39
6	female	35.1	39
7	male	36.4	40
8	male	38.2	38
	Mean	37.31	37.5

(c) For each student, the change from the predicted points to the final points ($f - p$) was calculated.

(i) Find the mean change.

(ii) Find the standard deviation of the changes. [3]

(d) Use a paired t-test to determine whether there is significant evidence that the students in school A have improved their IB points since the start of the course. [4]

(This question continues on the following page)

(Question 1 continued)

The data for school B is shown in the following table.

School B

Student number	Gender	Final IB points– Predicted IB points ($f-p$)
1	male	8.7
2	female	−1.1
3	female	4.8
4	female	−1.5
5	male	2.5
6	female	3.2
7	female	−1.3
8	female	3.1
	Mean	2.3

(e) (i) Use an appropriate test to determine whether there is evidence, at the 5% significance level, that the students in school B have improved more than those in school A.

(ii) State why it was important to test that both sets of points were normally distributed. [6]

(This question continues on the following page)

(Question 1 continued)

School A also gives each student a score for effort in each subject. This effort score is based on a scale of 1 to 5 where 5 is regarded as outstanding effort.

Student number	Gender	Predicted IB points	Final IB points	Average effort score
1	male	43.2	44	4.4
2	male	36.5	34	4.2
3	female	37.1	38	4.7
4	male	30.9	28	4.3
5	male	41.1	39	3.9
6	female	35.1	39	4.9
7	male	36.4	40	4.9
8	male	38.2	38	4.3
	Mean	37.31	37.5	4.45

It is claimed that the effort put in by a student is an important factor in improving upon their predicted IB points.

(f) (i) Perform a test on the data from school A to show it is reasonable to assume a linear relationship between effort scores and improvements in IB points. You may assume effort scores follow a normal distribution.

(ii) Hence, find the expected improvement between predicted and final points for an increase of one unit in effort grades, giving your answer to one decimal place. [4]

A mathematics teacher in school A claims that the comparison between the two schools is not valid because the sample for school B contained mainly girls and that for school A, mainly boys. She believes that girls are likely to show a greater improvement from their predicted points to their final points.

She collects more data from other schools, asking them to class their results into four categories as shown in the following table.

	$(f-p)<-2$	$-2\le(f-p)<0$	$0\le(f-p)<2$	$(f-p)\ge 2$
Male	6	8	10	9
Female	3	8	14	8

(g) Use an appropriate test to determine whether showing an improvement is independent of gender. [6]

(h) If you were to repeat the test performed in part (e) intending to compare the quality of the teaching between the two schools, suggest **two** ways in which you might choose your sample to improve the validity of the test. [2]

2. [Maximum mark: 28]

The number of brown squirrels, x, in an area of woodland can be modelled by the following differential equation.

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{x}{1000}(2000 - x)\text{, where } x > 0$$

(a) (i) Find the equilibrium population of brown squirrels suggested by this model.

(ii) Explain why the population of squirrels is increasing for values of x less than this value. [3]

One year conservationists notice that some black squirrels are moving into the woodland. The two species of squirrel are in competition for the same food supplies. Let y be the number of black squirrels in the woodland.

Conservationists wish to predict the likely future populations of the two species of squirrels. Research from other areas indicates that when the two populations come into contact the growth can be modelled by the following differential equations, in which t is measured in tens of years.

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{x}{1000}(2000 - x - 2y),\ x,\ y \geq 0$$
$$\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{y}{1000}(3000 - 3x - y),\ x,\ y \geq 0$$

An equilibrium point for the populations occurs when both $\frac{\mathrm{d}x}{\mathrm{d}t} = 0$ and $\frac{\mathrm{d}y}{\mathrm{d}t} = 0$.

(b) (i) Verify that $x = 800$, $y = 600$ is an equilibrium point.

(ii) Find the other three equilibrium points. [6]

(This question continues on the following page)

(Question 2 continued)

When the two populations are small the model can be reduced to the linear system

$$\frac{\mathrm{d}x}{\mathrm{d}t} = 2x$$
$$\frac{\mathrm{d}y}{\mathrm{d}t} = 3y\ .$$

(c) (i) By using separation of variables, show that the general solution of $\frac{\mathrm{d}x}{\mathrm{d}t} = 2x$ is $x = A\mathrm{e}^{2t}$.

(ii) Write down the general solution of $\frac{\mathrm{d}y}{\mathrm{d}t} = 3y$.

(iii) If both populations contain 10 squirrels at $t = 0$ use the solutions to parts (c) (i) and (ii) to estimate the number of black and brown squirrels when $t = 0.2$. Give your answers to the nearest whole numbers. [7]

For larger populations, the conservationists decide to use Euler's method to find the long-term outcomes for the populations. They will use Euler's method with a step length of 2 years ($t = 0.2$).

(d) (i) Write down the expressions for x_{n+1} and y_{n+1} that the conservationists will use.

(ii) Given that the initial populations are $x = 100$, $y = 100$, find the populations of each species of squirrel when $t = 1$.

(iii) Use further iterations of Euler's method to find the long-term population for each species of squirrel from these initial values.

(iv) Use the same method to find the long-term populations of squirrels when the initial populations are $x = 400$, $y = 100$. [7]

(e) Use Euler's method with step length 0.2 to sketch, on the same axes, the approximate trajectories for the populations with the following initial populations.

(i) $x = 1000$, $y = 1500$

(ii) $x = 1500$, $y = 1000$ [3]

(f) Given that the equilibrium point at $(800, 600)$ is a saddle point, sketch the phase portrait for $x \geq 0$, $y \geq 0$ on the same axes used in part (e). [2]

SPEC/5/MATAI/HP3/ENG/TZ0/XX/M

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

Specimen paper

Mathematics: applications and interpretation

Higher level

Paper 3

9 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

*Award marks using the annotations as noted in the markscheme eg **M1**, **A2**.*

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***M2***, ***A3***, *etc.*, do **not** split the marks, unless there is a note.
- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award the final ***A1***. An exception to this may be in numerical answers, where a correct exact value is followed by an incorrect decimal. However, if the incorrect decimal is carried through to a subsequent part, and correct ***FT*** working shown, award ***FT*** marks as appropriate but do not award the final ***A1*** in that part.

Examples

	Correct answer seen	Further working seen	Action
1.	$8\sqrt{2}$	$5.65685...$ *(incorrect decimal value)*	Award the final ***A1*** *(ignore the further working)*
2.	$\frac{1}{4}\sin 4x$	$\sin x$	Do not award the final ***A1***
3.	$\log a - \log b$	$\log(a-b)$	Do not award the final ***A1***

3 Implied marks

Implied marks appear in ***brackets e.g. (M1)****, and can only be awarded if* ***correct*** *work is seen or if implied in subsequent working.*

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

4 Follow through marks (only applied after an error is made)

*Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the answer (i.e. there is no working expected), then **FT** marks should be awarded if appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, use of $r>1$ for the sum of an infinite GP, $\sin\theta=1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- Exceptions to this rule will be explicitly noted on the markscheme.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.

5 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). Apply a **MR** penalty of 1 mark to that question*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta=1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates’ own work does **not** constitute a misread, it is an error.
- The ***MR*** penalty can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme

- Alternative methods for complete questions are indicated by ***METHOD 1***, ***METHOD 2***, *etc*.
- Alternative solutions for part-questions are indicated by ***EITHER*** . . . ***OR***.

7 Alternative forms

Unless the question specifies otherwise, ***accept*** *equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

8 Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. There are two types of accuracy errors, and the final answer mark should not be awarded if these errors occur.

- **Rounding errors**: only applies to final answers not to intermediate steps.
- **Level of accuracy**: when this is not specified in the question the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures*.

9 Calculators

A GDC is required for this examination, but calculators with symbolic manipulation features/ CAS functionality are not allowed.

Calculator notation
The subject guide says:
Students must always use correct mathematical notation, not calculator notation.

Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

1. (a) χ^2 (goodness of fit) ***A1***

[1 mark]

(b) ***EITHER***

because aim is to measure improvement

OR

because the students may be of different ability in the two schools ***R1***

[1 mark]

(c) (i) 0.1875 (accept 0.188, 0.19) ***A1***

(ii) 2.46 ***(M1)A1***

Note: Award ***(M1)A0*** for 2.63.

[3 marks]

(d) H_0: there has been no improvement

H_1: there has been an improvement ***A1***

attempt at a one-tailed paired t-test ***(M1)***

p-value $= 0.423$ ***A1***

there is no significant evidence that the students have improved ***R1***

Note: If the hypotheses are not stated award a maximum of ***A0M1A1R0***.

[4 marks]

(e) (i) H_0: there is no difference between the schools

H_1: school B did better than school A ***A1***

one-tailed 2 sample t-test ***(M1)***

p-value $= 0.0984$ ***A1***

$0.0984 > 0.05$ (not significant at the 5% level) so do not reject the null hypothesis ***R1A1***

Note: The final ***A1*** cannot be awarded following an incorrect reason.
The final ***R1A1*** can follow through from their incorrect p-value.
Award a maximum of ***A1(M1)A0R1A1*** for p-value $= 0.0993$.

(ii) sample too small for the central limit theorem to apply (and t-tests assume normal distribution) ***R1***

[6 marks]

continued…

Question 1 continued

(f) (i) $H_0: \rho = 0$

$H_1: \rho > 0$ ***A1***

Note: Allow hypotheses to be expressed in words.

p-value $= 0.00157$ ***A1***

$(0.00157 < 0.01)$ there is a significant evidence of a (linear) correlation between effort and improvement (so it is reasonable to assume a linear relationship) ***R1***

(ii) (gradient of line of regression =) 6.6 ***A1***

[4 marks]

(g) H_0: improvement and gender are independent

H_1: improvement and gender are not independent ***A1***

choice of χ^2 test for independence ***(M1)***

groups first two columns as expected values in first column less than 5 ***M1***

new observed table

	$(f-p)<0$	$0 \le (f-p) < 2$	$(f-p) \ge 2$
Male	14	10	9
Female	11	14	8

(A1)

p-value $= 0.581$ ***A1***

no significant evidence that gender and improvement are dependent ***R1***

[6 marks]

(h) *For example:*
larger samples / include data from whole school
take equal numbers of boys and girls in each sample
have a similar range of abilities in each sample
(if possible) have similar ranges of effort ***R1R1***

Note: Award ***R1*** for each reasonable suggestion to improve the validity of the test.

[2 marks]

Total [27 marks]

2. (a) (i) 2000 ***(M1)A1***

(ii) because the value of $\frac{\mathrm{d}x}{\mathrm{d}t}$ is positive (for $x > 0$) ***R1***

[3 marks]

(b) (i) substitute $x = 800$, $y = 600$ into both equations ***M1***

both equations equal 0 ***A1***

hence an equilibrium point ***AG***

(ii) $x = 0$, $y = 0$ ***A1***

$x = 2000$, $y = 0$, $x = 0$, $y = 3000$ ***M1A1A1***

Note: Award ***M1*** for an attempt at solving the system provided some values of x and y are found.

[6 marks]

(c) (i) $\int \frac{1}{x}\,\mathrm{d}x = \int 2\,\mathrm{d}t$ ***M1***

$\ln x = 2t + c$ ***A1A1***

Note: Award ***A1*** for RHS, ***A1*** for LHS.

$x = \mathrm{e}^{c}\mathrm{e}^{2t}$ ***M1***

$x = A\mathrm{e}^{2t}$ $\left(\text{where } A = \mathrm{e}^{c}\right)$ ***AG***

(ii) $y = B\mathrm{e}^{3t}$ ***A1***

Note: Allow any letter for the constant term, including A.

(iii) $x = 15$, $y = 18$ ***(M1)A1***

[7 marks]

continued…

Question 2 continued

(d) (i) $x_{n+1} = x_n + 0.2\dfrac{x_n}{1000}(2000 - x_n - 2y_n)$

$y_{n+1} = y_n + 0.2\dfrac{y_n}{1000}(3000 - 3x_n - y_n)$ ***M1A1***

Note: Accept equivalent forms.

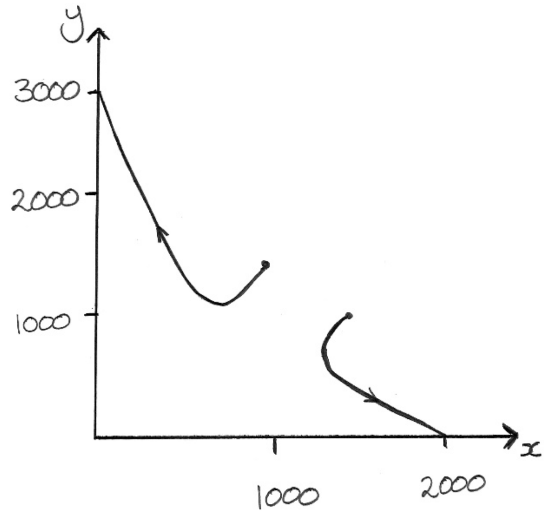
(ii) $x = 319$, $y = 617$ ***(M1)A1A1***

(iii) number of brown squirrels go down to 0, black squirrels to a population of 3000 ***A1***

(iv) number of brown squirrels go to 2000, number of black squirrels goes down to 0 ***A1***

[7 marks]

(e) (i) ***AND*** (ii)



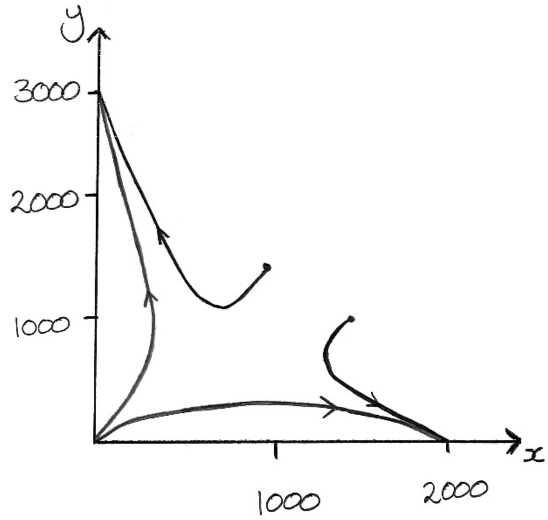
M1A1A1

[3 marks]

continued…

Question 2 continued

(f)



A1A1

Note:	Award ***A1*** for a trajectory beginning close to $(0, 0)$ and going to $(0, 3000)$ and ***A1*** for a trajectory beginning close to $(0, 0)$ and going to $(2000, 0)$ in approximately the correct places.

[2 marks]

Total [28 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Standard level
Paper 1

Specimen paper

Candidate session number

1 hour 30 minutes

Instructions to candidates

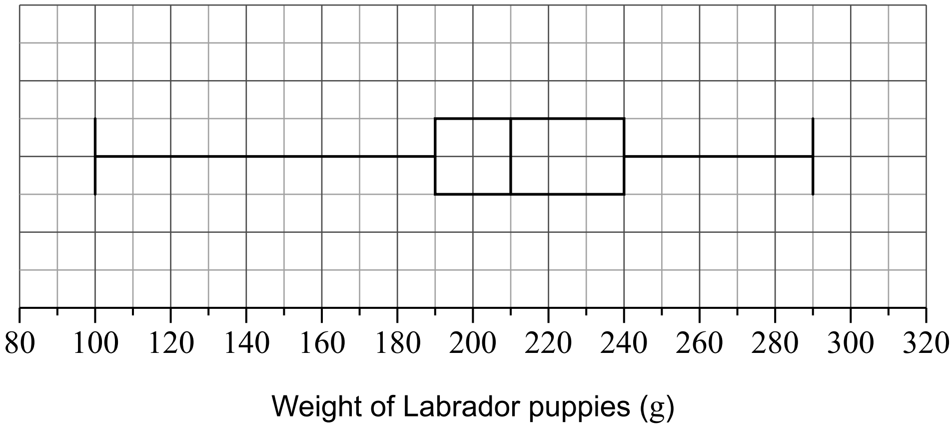
- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 6]

Palvinder breeds Labrador puppies at his farm. Over many years he recorded the weight (g) of the puppies.

The data is illustrated in the following box and whisker diagram.



(a) Write down the median weight of the puppies. [1]

(b) Write down the upper quartile. [1]

(c) Find the interquartile range. [2]

The weights of these Labrador puppies are normally distributed.

(d) Find the weight of the heaviest possible puppy that is not an outlier. [2]

(This question continues on the following page)

(Question 1 continued)

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2. [Maximum mark: 6]

The Osaka Tigers basketball team play in a multilevel stadium.

The most expensive tickets are in the first row. The ticket price, in Yen (¥), for each row forms an arithmetic sequence. Prices for the first three rows are shown in the following table.

Ticket pricing per game	
1st row	6800 Yen
2nd row	6550 Yen
3rd row	6300 Yen

(a) Write down the value of the common difference, d [1]

(b) Calculate the price of a ticket in the 16th row. [2]

(c) Find the total cost of buying 2 tickets in each of the first 16 rows. [3]

3. [Maximum mark: 6]

At the end of a school day, the Headmaster conducted a survey asking students in how many classes they had used the internet.

The data is shown in the following table.

Number of classes in which the students used the internet	0	1	2	3	4	5	6
Number of students	20	24	30	k	10	3	1

(a) State whether the data is discrete or continuous. [1]

The mean number of classes in which a student used the internet is 2.

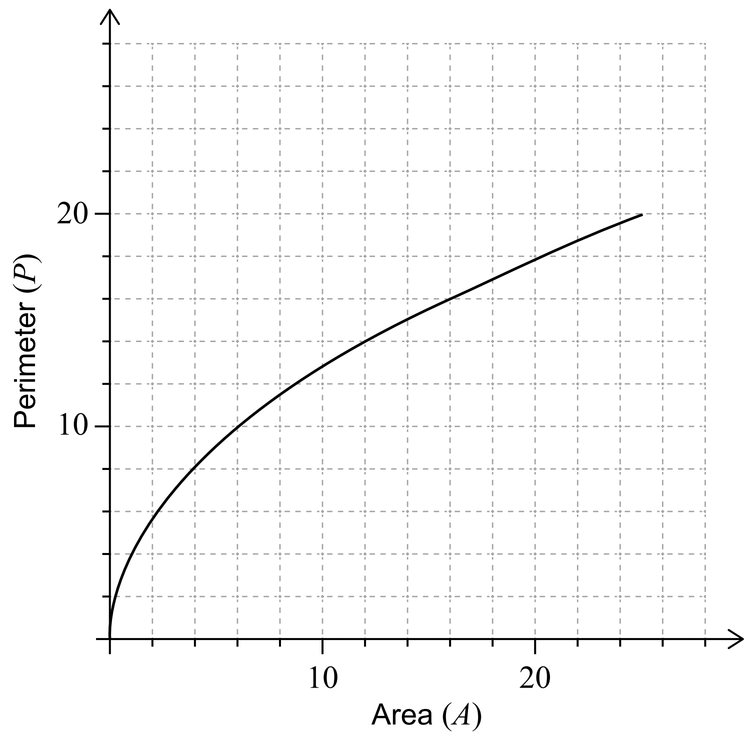
(b) Find the value of k. [4]

It was not possible to ask every person in the school, so the Headmaster arranged the student names in alphabetical order and then asked every 10th person on the list.

(c) Identify the sampling technique used in the survey. [1]

4. [Maximum mark: 6]

The perimeter of a given square P can be represented by the function $P(A) = 4\sqrt{A}$, $A \geq 0$, where A is the area of the square. The graph of the function P is shown for $0 \leq A \leq 25$.



(a) Write down the value of $P(25)$. [1]

The range of $P(A)$ is $0 \leq P(A) \leq n$.

(b) Hence write down the value of n . [1]

(c) On the axes above, draw the graph of the inverse function, P^{-1} . [3]

(d) In the context of the question, explain the meaning of $P^{-1}(8) = 4$. [1]

(This question continues on the following page)

(Question 4 continued)

5. [Maximum mark: 6]

Professor Vinculum investigated the migration season of the Bulbul bird from their natural wetlands to a warmer climate.

He found that during the migration season their population, P could be modelled by $P = 1350 + 400(1.25)^{-t}$, $t \geq 0$, where t is the number of days since the start of the migration season.

(a) Find the population of the Bulbul birds,

(i) at the start of the migration season.

(ii) in the wetlands after 5 days. [3]

(b) Calculate the time taken for the population to decrease below 1400. [2]

(c) According to this model, find the smallest possible population of Bulbul birds during the migration season. [1]

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6. [Maximum mark: 5]

As part of a study into healthy lifestyles, Jing visited Surrey Hills University. Jing recorded a person's position in the university and how frequently they ate a salad. Results are shown in the table.

	Salad meals per week			
	0	**1 – 2**	**3 – 4**	**> 4**
Students	45	26	18	6
Professors	15	8	5	12
Staff and Administration	16	13	10	6

Jing conducted a χ^2 test for independence at a 5% level of significance.

(a) State the null hypothesis. [1]

(b) Calculate the p-value for this test. [2]

(c) State, giving a reason, whether the null hypothesis should be accepted. [2]

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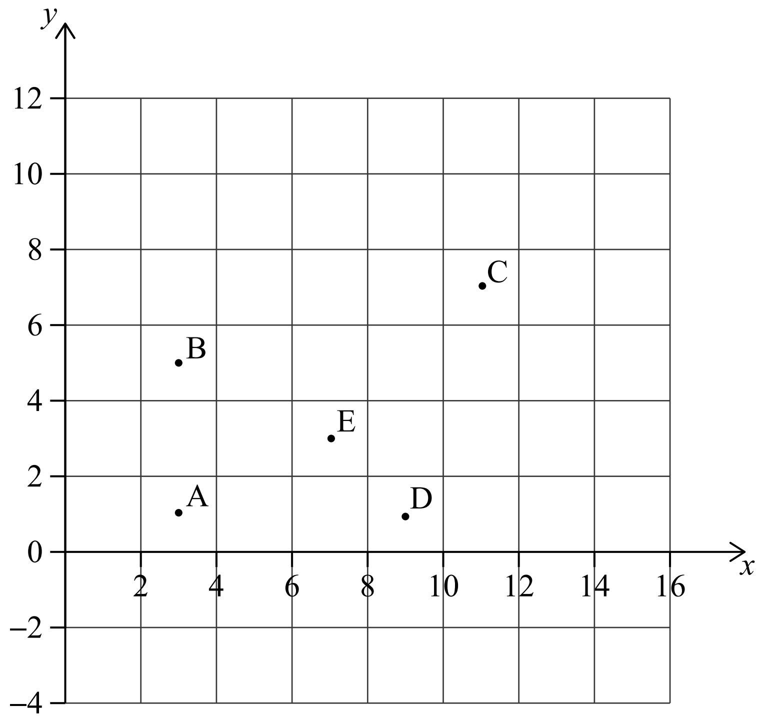
7. [Maximum mark: 6]

Points $A(3, 1)$, $B(3, 5)$, $C(11, 7)$, $D(9, 1)$ and $E(7, 3)$ represent snow shelters in the Blackburn National Forest. These snow shelters are illustrated in the following coordinate axes.

Horizontal scale: 1 unit represents $1\,\text{km}$.

Vertical scale: 1 unit represents $1\,\text{km}$.

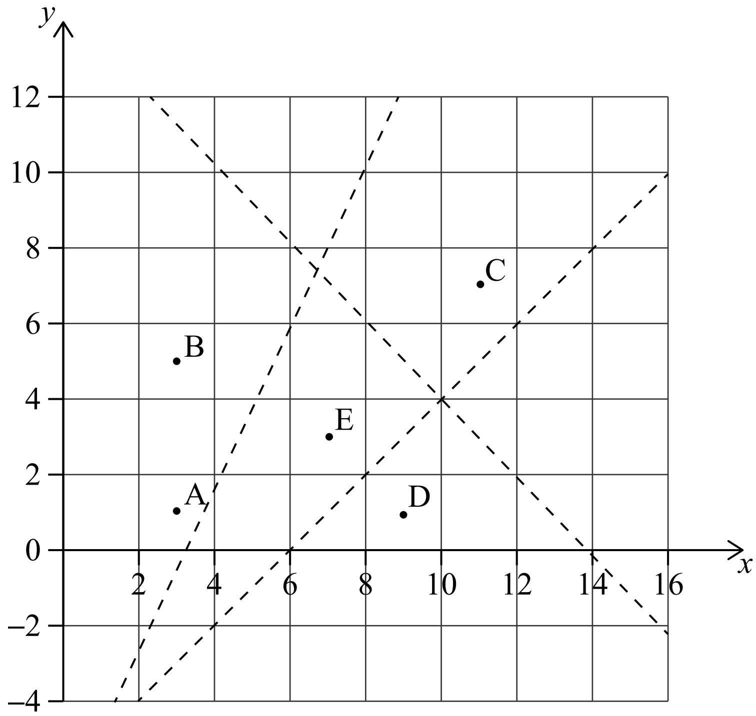


(a) Calculate the gradient of the line segment AE. [2]

(This question continues on the following page)

(Question 7 continued)

The Park Ranger draws three straight lines to form an incomplete Voronoi diagram.



(b) Find the equation of the line which would complete the Voronoi cell containing site E. Give your answer in the form $ax + by + d = 0$ where $a, b, d \in \mathbb{Z}$. [3]

(c) In the context of the question, explain the significance of the Voronoi cell containing site E. [1]

8. [Maximum mark: 4]

The intensity level of sound, L measured in decibels (dB), is a function of the sound intensity, S watts per square metre $(\mathrm{W\,m^{-2}})$. The intensity level is given by the following formula.

$$L = 10\log_{10}(S \times 10^{12}),\ S \geq 0$$

(a) An orchestra has a sound intensity of $6.4 \times 10^{-3}\,\mathrm{W\,m^{-2}}$. Calculate the intensity level, L of the orchestra. [2]

(b) A rock concert has an intensity level of $112\,\mathrm{dB}$. Find the sound intensity, S. [2]

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9. [Maximum mark: 6]

Ms Calhoun measures the heights of students in her mathematics class. She is interested to see if the mean height of male students, μ_1, is the same as the mean height of female students, μ_2. The information is recorded in the table.

Male height (cm)	150	148	143	152	151	149	147	
Female height (cm)	148	152	154	147	146	153	152	150

At the 10% level of significance, a t-test was used to compare the means of the two groups. The data is assumed to be normally distributed and the standard deviations are equal between the two groups.

(a) (i) State the null hypothesis.

(ii) State the alternative hypothesis. [2]

(b) Calculate the p-value for this test. [2]

(c) State, giving a reason, whether Ms Calhoun should accept the null hypothesis. [2]

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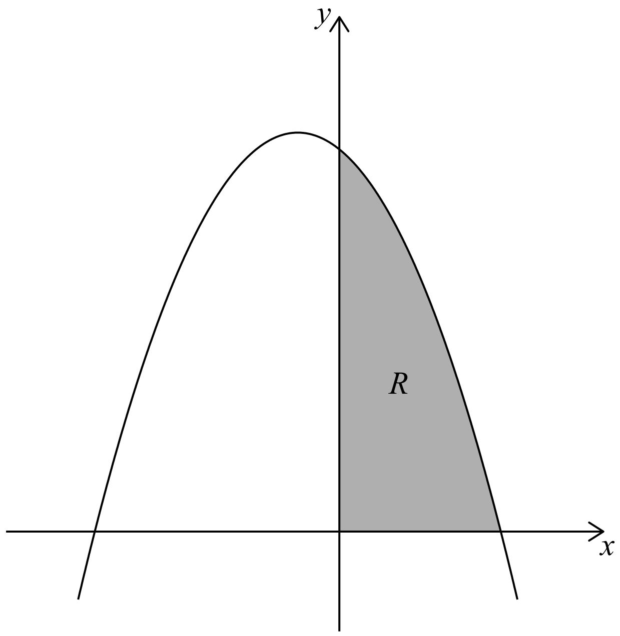
10. [Maximum mark: 5]

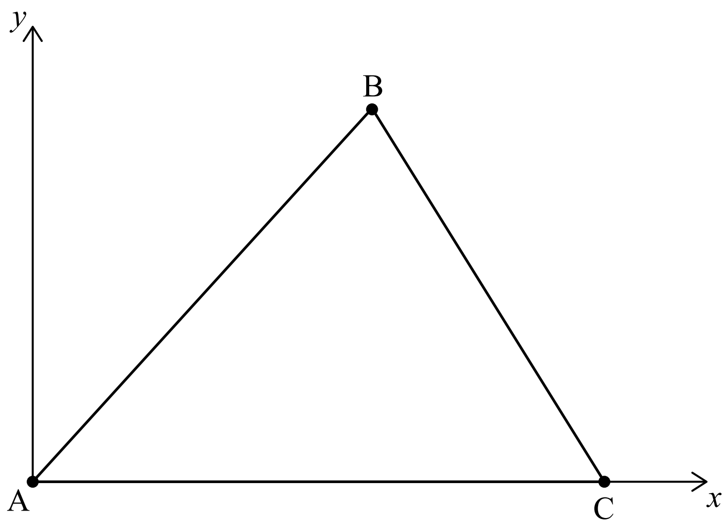
The following diagram shows part of the graph of $f(x) = (6 - 3x)(4 + x)$, $x \in \mathbb{R}$. The shaded region R is bounded by the x-axis, y-axis and the graph of f.



(a) Write down an integral for the area of region R. [2]

(b) Find the area of region R. [1]

The three points $\mathrm{A}(0, 0)$, $\mathrm{B}(3, 10)$ and $\mathrm{C}(a, 0)$ define the vertices of a triangle.



(c) Find the value of a, the x-coordinate of C, such that the area of the triangle is equal to the area of region R. [2]

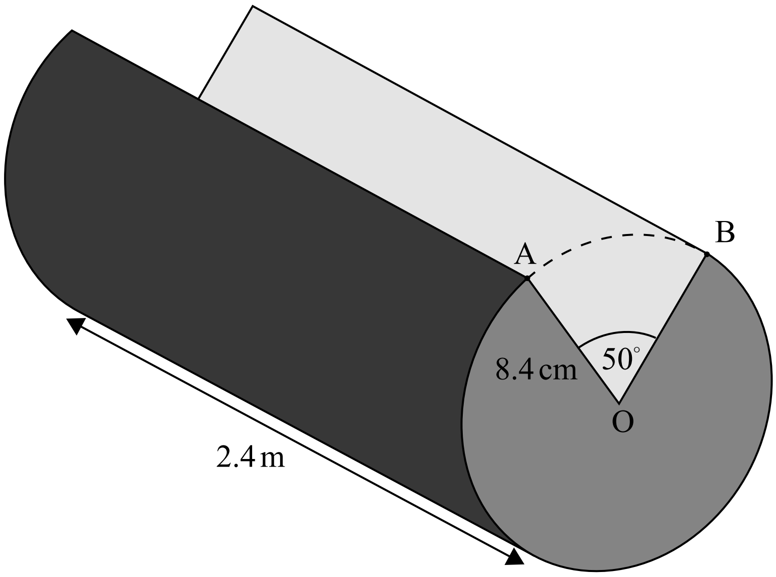
(This question continues on the following page)

(Question 10 continued)

11. [Maximum mark: 4]

Helen is building a cabin using cylindrical logs of length 2.4 m and radius 8.4 cm. A wedge is cut from one log and the cross-section of this log is illustrated in the following diagram.

diagram not to scale



Find the volume of this log.

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12. [Maximum mark: 6]

Jae Hee plays a game involving a biased six-sided die.
The faces of the die are labelled -3, -1, 0, 1, 2 and 5.
The score for the game, X, is the number which lands face up after the die is rolled.
The following table shows the probability distribution for X.

Score x	-3	-1	0	1	2	5
$\mathbf{P}(X = x)$	$\frac{1}{18}$	p	$\frac{3}{18}$	$\frac{1}{18}$	$\frac{2}{18}$	$\frac{7}{18}$

(a) Find the exact value of p. [1]

Jae Hee plays the game once.

(b) Calculate the expected score. [2]

Jae Hee plays the game twice and adds the two scores together.

(c) Find the probability Jae Hee has a **total** score of -3. [3]

13. [Maximum mark: 6]

Mr Burke teaches a mathematics class with 15 students. In this class there are 6 female students and 9 male students.

Each day Mr Burke randomly chooses one student to answer a homework question.

(a) Find the probability that on any given day Mr Burke chooses a female student to answer a question. [1]

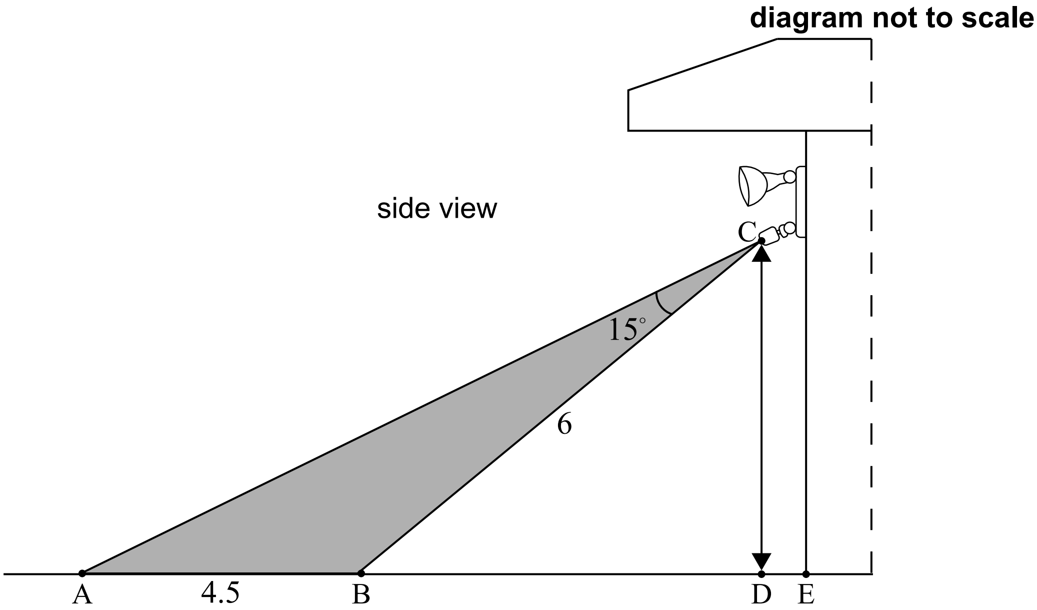
In the first month, Mr Burke will teach his class 20 times.

(b) Find the probability he will choose a female student 8 times. [2]

(c) Find the probability he will choose a male student at most 9 times. [3]

14. [Maximum mark: 8]

Ollie has installed security lights on the side of his house that are activated by a sensor. The sensor is located at point C directly above point D. The area covered by the sensor is shown by the shaded region enclosed by triangle ABC. The distance from A to B is $4.5\,\mathrm{m}$ and the distance from B to C is $6\,\mathrm{m}$. Angle $\mathrm{A\hat{C}B}$ is 15°.



(a) Find $\mathrm{C\hat{A}B}$. [3]

Point B on the ground is $5\,\mathrm{m}$ from point E at the entrance to Ollie's house. He is $1.8\,\mathrm{m}$ tall and is standing at point D, below the sensor. He walks towards point B.

(b) Find the distance Ollie is **from the entrance to his house** when he first activates the sensor. [5]

Please **do not** write on this page.

Answers written on this page
will not be marked.

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

Specimen paper

Mathematics: applications and interpretation

Standard level

Paper 1

13 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

*Award marks using the annotations as noted in the markscheme eg **M1**, **A2**.*

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***M2***, ***A3***, *etc*., do **not** split the marks, unless there is a note.
- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award the final ***A1***. An exception to this may be in numerical answers, where a correct exact value is followed by an incorrect decimal. However, if the incorrect decimal is carried through to a subsequent part, and correct ***FT*** working shown, award ***FT*** marks as appropriate but do not award the final ***A1*** in that part.

Examples

	Correct answer seen	**Further working seen**	**Action**
1.	$8\sqrt{2}$	$5.65685...$ *(incorrect decimal value)*	Award the final ***A1*** *(ignore the further working)*
2.	$\frac{1}{4}\sin 4x$	$\sin x$	Do not award the final ***A1***
3.	$\log a - \log b$	$\log(a-b)$	Do not award the final ***A1***

3 Implied marks

Implied marks appear in ***brackets e.g. (M1)****, and can only be awarded if* ***correct*** *work is seen or if implied in subsequent working.*

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

4 Follow through marks (only applied after an error is made)

*Follow through (****FT****) marks are awarded where an incorrect answer from one* ***part*** *of a question is used correctly in* ***subsequent*** *part(s) or subpart(s). Usually, to award* ***FT*** *marks,* ***there must be working present*** *and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the answer (i.e. there is no working expected), then* ***FT*** *marks should be awarded if appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- Exceptions to this rule will be explicitly noted on the markscheme.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.

5 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (****MR****). Apply a* ***MR*** *penalty of 1 mark to that question*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates’ own work does **not** constitute a misread, it is an error.
- The ***MR*** penalty can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme

- Alternative methods for complete questions are indicated by ***METHOD 1***, ***METHOD 2***, *etc*.
- Alternative solutions for part-questions are indicated by ***EITHER*** . . . ***OR***.

7 Alternative forms

Unless the question specifies otherwise, ***accept*** *equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

8 Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. There are two types of accuracy errors, and the final answer mark should not be awarded if these errors occur.

- **Rounding errors**: only applies to final answers not to intermediate steps.
- **Level of accuracy**: when this is not specified in the question the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures*.

9 Calculators

A GDC is required for this examination, but calculators with symbolic manipulation features/ CAS functionality are not allowed.

Calculator notation

The subject guide says:

Students must always use correct mathematical notation, not calculator notation.

Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

1. (a) 210g ***A1***

[1 mark]

(b) 240g ***A1***

[1 mark]

(c) $240-190$ ***(M1)***

$=50\text{g}$ ***A1***

[2 marks]

(d) $240 + 1.5 \times (50)$ ***M1***

$=315\text{g}$ ***A1***

[2 marks]

Total [6 marks]

2. (a) $(d =) - 250$ ***A1***

[1 mark]

(b) $(u_{16} =) 6800 + (16 - 1)(- 250)$ ***M1***

$(¥) 3050$ ***A1***

[2 marks]

(c) $(S_{16} =)\left(\frac{16}{2}\right)\left(2 \times 6800 + (16 - 1)(- 250)\right) \times 2$ ***M1M1***

Note: Award ***M1*** for correct substitution into arithmetic series formula. Award ***M1*** for multiplication by 2 seen.

OR

$(S_{16} =)\left(\frac{16}{2}\right)(6800 + 3050) \times 2$ ***M1M1***

Note: Award ***M1*** for correct substitution into arithmetic series formula. Award ***M1*** for multiplication by 2 seen.

$(¥) 158\,000$ $(157\,600)$ ***A1***

[3 marks]

Total [6 marks]

3. (a) discrete *A1*

[1 mark]

(b) $\dfrac{24 + 60 + 3k + 40 + 15 + 6}{88 + k} = 2$ ***M1A1***

> **Note:** Award ***M1*** for substitution into the formula for the mean, award ***A1*** for a correct equation.

attempt to solve their equation ***(M1)***

$k = 31$ ***A1***

[4 marks]

(c) systematic ***A1***

[1 mark]

Total [6 marks]

4. (a) 20 ***A1***

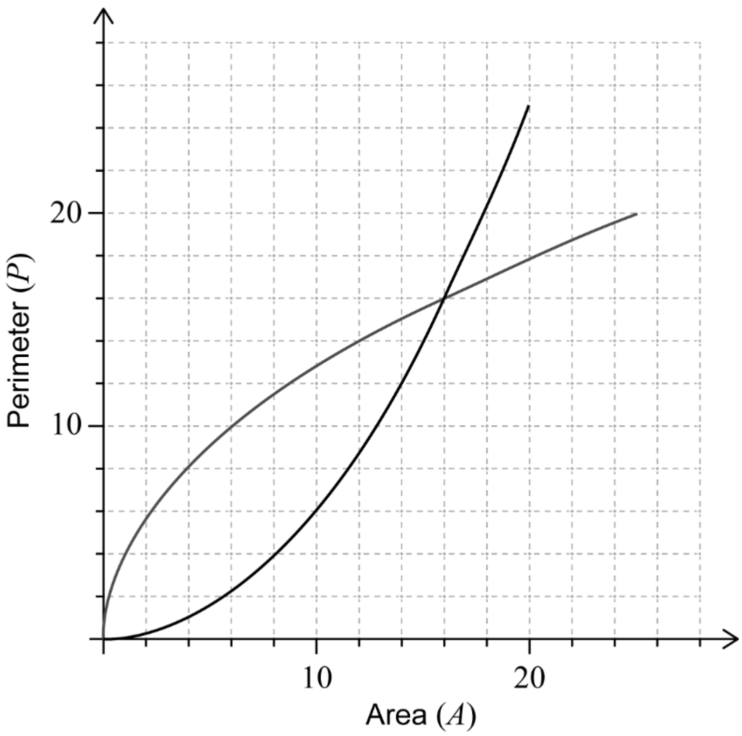
[1 mark]

(b) $n = 20$ ***A1***

Note: Follow through from part (a).

[1 mark]

(c)



(M1)A1A1

Note: Award ***(M1)*** for reflection in the line $P = A$, award ***A1*** for endpoint at $(20, 25)$, award ***A1*** for passing through $(16, 16)$.

[3 marks]

(d) when the perimeter is 8, the area is 4 ***A1***

[1 mark]

Total [6 marks]

5. (a) (i) 1750 ***A1***

(ii) $1350 + 400\,(1.25)^{-5}$ ***(M1)***

$= 1480$ ***A1***

Note: Accept 1481.

[3 marks]

(b) $1400 = 1350 + 400(1.25)^{-t}$ ***(M1)***

9.32 (days ($9.31885\ldots$) (days)) ***A1***

[2 marks]

(c) 1350 ***A1***

Note: Accept 1351 as a valid interpretation of the model as $P = 1350$ is an asymptote.

[1 mark]

Total [6 marks]

6. (a) number of salad meals per week is independent of a person's position in the university ***A1***

Note: Accept "not associated" instead of independent.

[1 mark]

(b) 0.0201 ($0.0201118\ldots$) ***A2***

[2 marks]

(c) $0.0201 < 0.05$ ***R1***

the null hypothesis is rejected ***A1***

[2 marks]

Note: Award ***(R1)*** for a correct comparison of their p-value to the test level, award ***(A1)*** for the correct interpretation from that comparison. Do not award ***(R0)(A1)***.

Total [5 marks]

7. (a) $\frac{3-1}{7-3}$ ***(M1)***

$=0.5$ ***A1***

[2 marks]

(b) $y-2=-2(x-5)$ ***(A1)(M1)***

Note: Award ***(A1)*** for their -2 seen, award ***(M1)*** for the correct substitution of $(5, 2)$ and their normal gradient in equation of a line.

$2x+y-12=0$ ***A1***

[3 marks]

(c) every point in the cell is closer to E than any other snow shelter ***A1***

[1 mark]

Total [6 marks]

8. (a) $10\log_{10}\left(6.4\times10^{-3}\times10^{12}\right)$ ***(M1)***

$=98.1(\mathrm{dB})\ (98.06179\ldots)$ ***A1***

[2 marks]

(b) $112=10\log_{10}\left(S\times10^{12}\right)$ ***(M1)***

$0.158\left(\mathrm{W\,m^{-2}}\right)\left(0.158489\ldots\left(\mathrm{W\,m^{-2}}\right)\right)$ ***A1***

[2 marks]

Total [4 marks]

9. (a) (i) $\mu_1 - \mu_2 = 0$ ***A1***

(ii) $\mu_1 - \mu_2 \neq 0$ ***A1***

Note: Accept equivalent statements in words.

[2 marks]

(b) $0.296\ (0.295739\ldots)$ ***A2***

[2 marks]

(c) $0.296 > 0.1$ ***R1***

fail to reject the null hypothesis, there is no difference between the mean height of male and female students ***A1***

Note: Award ***(R1)*** for a correct comparison of their p-value to the test level, award ***(A1)*** for the correct interpretation from that comparison. Do not award ***R0A1***.

[2 marks]

Total [6 marks]

10. (a) $A = \int_0^2 (6-3x)(4+x)\mathrm{d}x$ ***A1A1***

Note: Award ***A1*** for the limits $x = 0,\ x = 2$. Award ***A1*** for an integral of $f(x)$.

[2 marks]

(b) 28 ***A1***

[1 mark]

(c) $28 = 0.5 \times a \times 10$ ***M1***

$5.6\left(\frac{28}{5}\right)$ ***A1***

[2 marks]

Total [5 marks]

11. volume $= 240\left(\pi \times 8.4^2 - \frac{1}{2} \times 8.4^2 \times \frac{50 \times \pi}{180}\right)$ ***M1M1M1***

Note: Award ***M1*** $240 \times \text{area}$, award ***M1*** for correctly substituting area sector formula, award ***M1*** for subtraction of their area of the sector from area of circle.

$= 45800\ (= 45811.96071)$ ***A1***

Total [4 marks]

12. (a) $\frac{4}{18}\left(\frac{2}{9}\right)$ ***A1***

[1 mark]

(b) $-3 \times \frac{1}{18} + (-1) \times \frac{4}{18} + 0 \times \frac{3}{18} + \ldots + 5 \times \frac{7}{18}$ ***(M1)***

Note: Award ***(M1)*** for their correct substitution into the formula for expected value.

$= 1.83\left(\frac{33}{18}, 1.83333\ldots\right)$ ***A1***

[2 marks]

(c) $2 \times \frac{1}{18} \times \frac{3}{18}$ ***(M1)(M1)***

Note: Award ***(M1)*** for $\frac{1}{18} \times \frac{3}{18}$, award ***(M1)*** for multiplying their product by 2.

$= \frac{1}{54}\left(\frac{6}{324}, 0.0185185\ldots, 1.85\%\right)$ ***A1***

[3 marks]

Total [6 marks]

13. (a) $\frac{6}{15}\left(0.4, \frac{2}{5}\right)$ ***A1***

[1 mark]

(b) $\mathrm{P}(X = 8)$ ***(M1)***

Note: Award ***(M1)*** for evidence of recognizing binomial probability. *eg*, $\mathrm{P}(X = 8)$, $X \sim \mathrm{B}\left(20, \frac{6}{15}\right)$.

$0.180\ (0.179705\ldots)$ ***A1***

[2 marks]

(c) $\mathrm{P}(\text{male}) = \frac{9}{15}\ (0.6)$ ***A1***

$\mathrm{P}(X \le 9) = 0.128\ (0.127521\ldots)$ ***(M1)A1***

Note: Award ***(M1)*** for evidence of correct approach *eg*, $\mathrm{P}(X \le 9)$.

[3 marks]

Total [6 marks]

14. (a) $\dfrac{\sin \text{C}\hat{\text{A}}\text{B}}{6} = \dfrac{\sin 15^\circ}{4.5}$ ***(M1)(A1)***

$\text{C}\hat{\text{A}}\text{B} = 20.2^\circ \ (20.187415\ldots)$ ***A1***

> **Note:** Award ***(M1)*** for substituted sine rule formula and award ***(A1)*** for correct substitutions.

[3 marks]

(b) $\text{C}\hat{\text{B}}\text{D} = 20.2 + 15 = 35.2^\circ$ ***A1***

(let X be the point on BD where Ollie activates the sensor)

$\tan 35.18741\ldots^\circ = \dfrac{1.8}{\text{BX}}$ ***(M1)***

> **Note:** Award ***A1*** for their correct angle $\text{C}\hat{\text{B}}\text{D}$. Award ***M1*** for correctly substituted trigonometric formula.

$\text{BX} = 2.55285\ldots$ ***A1***

$5 - 2.55285\ldots$ ***(M1)***

$= 2.45\,(\text{m})\ (2.44714\ldots)$ ***A1***

[5 marks]

Total [8 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Standard level
Paper 2

Specimen paper

1 hour 30 minutes

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

Blank page

Answer **all** questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 17]

In this question, give all answers to two decimal places.

Bryan decides to purchase a new car with a price of €$14\,000$, but cannot afford the full amount. The car dealership offers two options to finance a loan.

Finance option A:

A 6 year loan at a nominal annual interest rate of 14% **compounded quarterly**. No deposit required and repayments are made each quarter.

(a) (i) Find the repayment made each quarter.

(ii) Find the total amount paid for the car.

(iii) Find the interest paid on the loan. [7]

Finance option B:

A 6 year loan at a nominal annual interest rate of $r\,\%$ **compounded monthly**. Terms of the loan require a 10% deposit and monthly repayments of €250.

(b) (i) Find the amount to be borrowed for this option.

(ii) Find the annual interest rate, r. [5]

(c) State which option Bryan should choose. Justify your answer. [2]

Bryan's car depreciates at an annual rate of 25% per year.

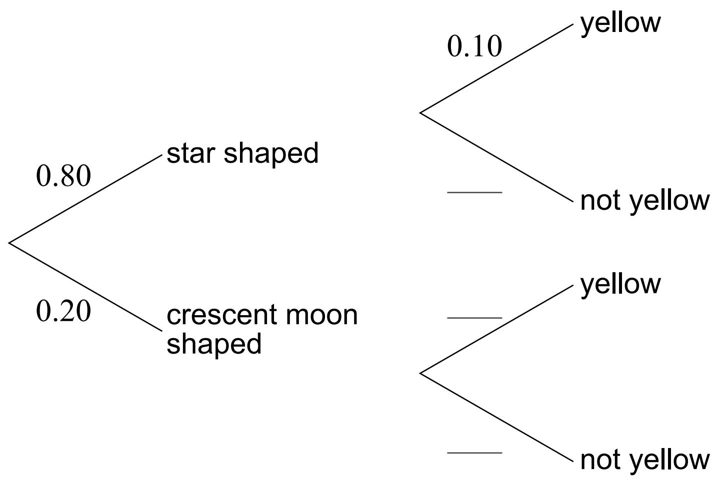
(d) Find the value of Bryan's car six years after it is purchased. [3]

2. [Maximum mark: 14]

Slugworth Candy Company sell a variety pack of colourful, shaped sweets.

The sweets are produced such that 80% are star shaped and 20% are shaped like a crescent moon. It is known that 10% of the stars and 30% of the crescent moons are coloured yellow.

(a) Using the given information, **copy** and complete the following tree diagram. [2]



(b) A sweet is selected at random.

(i) Find the probability that the sweet is yellow.

(ii) Given that the sweet is yellow, find the probability it is star shaped. [4]

(This question continues on the following page)

(Question 2 continued)

According to manufacturer specifications, the colours in each variety pack should be distributed as follows.

Colour	Brown	Red	Green	Orange	Yellow	Purple
Percentage (%)	15	25	20	20	10	10

Mr Slugworth opens a pack of 80 sweets and records the frequency of each colour.

Colour	Brown	Red	Green	Orange	Yellow	Purple
Observed Frequency	10	20	16	18	12	4

To investigate if the sample is consistent with manufacturer specifications, Mr Slugworth conducts a χ^2 goodness of fit test. The test is carried out at a 5% significance level.

(c) Write down the null hypothesis for this test. [1]

(d) **Copy** and complete the following table in your answer booklet. [2]

Colour	Brown	Red	Green	Orange	Yellow	Purple
Expected Frequency						

(e) Write down the number of degrees of freedom. [1]

(f) Find the p-value for the test. [2]

(g) State the conclusion of the test. Give a reason for your answer. [2]

3. [Maximum mark: 17]

The Malvern Aquatic Center hosted a 3 metre spring board diving event. The judges, Stan and Minsun awarded 8 competitors a score out of 10. The raw data is collated in the following table.

Competitors	A	B	C	D	E	F	G	H
Stan's score (x)	4.1	3	4.3	6	7.1	6	7.5	6
Minsun's score (y)	4.7	4.6	4.8	7.2	7.8	9	9.5	7.2

(a) (i) Write down the value of the Pearson's product–moment correlation coefficient, r.

(ii) Using the value of r, interpret the relationship between Stan's score and Minsun's score. [4]

(b) Write down the equation of the regression line y on x. [2]

(c) (i) Use your regression equation from part (b) to estimate Minsun's score when Stan awards a perfect 10.

(ii) State whether this estimate is reliable. Justify your answer. [4]

The Commissioner for the event would like to find the Spearman's rank correlation coefficient.

(d) **Copy** and complete the information in the following table. [2]

Competitors	A	B	C	D	E	F	G	H
Stan's Rank		8					1	4
Minsun's Rank		8					1	4.5

(e) (i) Find the value of the Spearman's rank correlation coefficient, r_s.

(ii) Comment on the result obtained for r_s. [4]

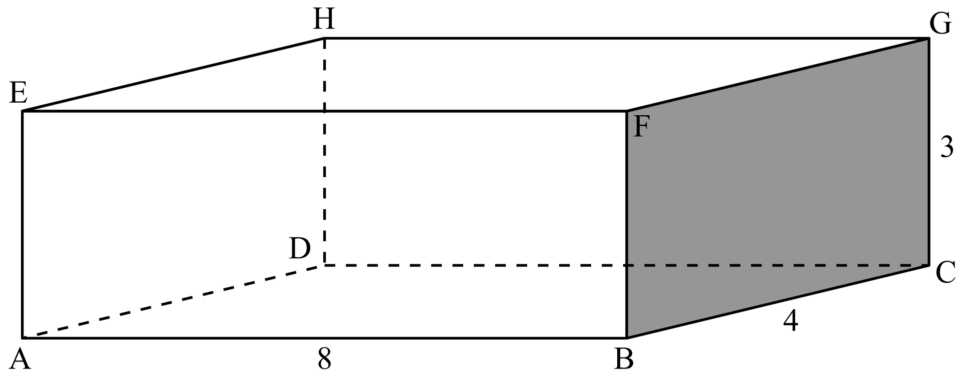
The Commissioner believes Minsun's score for competitor G is too high and so decreases the score from 9.5 to 9.1.

(f) Explain why the value of the Spearman's rank correlation coefficient r_s does not change. [1]

4. [Maximum mark: 15]

The Happy Straw Company manufactures drinking straws.

The straws are packaged in small closed rectangular boxes, each with length $8\,\text{cm}$, width $4\,\text{cm}$ and height $3\,\text{cm}$. The information is shown in the diagram.



(a) Calculate the surface area of the box in cm^2. [2]

(b) Calculate the length AG. [2]

Each week, the Happy Straw Company sells x boxes of straws. It is known that $\frac{dP}{dx} = -2x + 220$, $x \geq 0$, where P is the weekly profit, in dollars, from the sale of x thousand boxes.

(c) Find the number of boxes that should be sold each week to maximize the profit. [3]

The profit from the sale of $20\,000$ boxes is $\$1700$.

(d) Find $P(x)$. [5]

(e) Find the least number of boxes which must be sold each week in order to make a profit. [3]

5. [Maximum mark: 17]

The braking distance of a vehicle is defined as the distance travelled from where the brakes are applied to the point where the vehicle comes to a complete stop.

The speed, $s\,\mathrm{m\,s^{-1}}$, and braking distance, $d\,\mathrm{m}$, of a truck were recorded. This information is summarized in the following table.

Speed, s m s^{-1}	0	6	10
Braking distance, d m	0	12	60

This information was used to create Model A, where d is a function of s, $s \geq 0$.

Model A: $d(s) = ps^2 + qs$, where p, $q \in \mathbb{Z}$

At a speed of $6\,\mathrm{m\,s^{-1}}$, Model A can be represented by the equation $6p + q = 2$.

(a) (i) Write down a second equation to represent Model A, when the speed is $10\,\mathrm{m\,s^{-1}}$.

(ii) Find the values of p and q [4]

(b) Find the coordinates of the vertex of the graph of $y = d(s)$. [2]

(c) Using the values in the table and your answer to part (b), sketch the graph of $y = d(s)$ for $0 \leq s \leq 10$ and $-10 \leq d \leq 60$, clearly showing the vertex. [3]

(d) Hence, identify why Model A may not be appropriate at lower speeds. [1]

Additional data was used to create Model B, **a revised model** for the braking distance of a truck.

Model B: $d(s) = 0.95s^2 - 3.92s$

(e) Use Model B to calculate an estimate for the braking distance at a speed of $20\,\mathrm{m\,s^{-1}}$. [2]

The actual braking distance at $20\,\mathrm{m\,s^{-1}}$ is $320\,\mathrm{m}$.

(f) Calculate the percentage error in the estimate in part (e). [2]

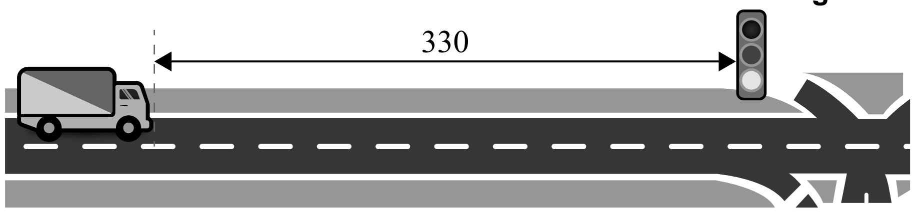
(This question continues on the following page)

(Question 5 continued)

It is found that once a driver realizes the need to stop their vehicle, 1.6 seconds will elapse, on average, before the brakes are engaged. During this reaction time, the vehicle will continue to travel at its original speed.

A truck approaches an intersection with speed $s\ \mathrm{m\,s^{-1}}$. The driver notices the intersection's traffic lights are red and they must stop the vehicle within a distance of $330\ \mathrm{m}$.

diagram not to scale



(g) Using model B and taking reaction time into account, calculate the maximum possible speed of the truck if it is to stop before the intersection. [3]

Markscheme

Specimen paper

Mathematics: applications and interpretation

Standard level

Paper 2

12 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme eg ***M1***, ***A2***.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***M2***, ***A3***, *etc.*, do **not** split the marks, unless there is a note.
- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award the final ***A1***. An exception to this may be in numerical answers, where a correct exact value is followed by an incorrect decimal. However, if the incorrect decimal is carried through to a subsequent part, and correct ***FT*** working shown, award ***FT*** marks as appropriate but do not award the final ***A1*** in that part.

Examples

	Correct answer seen	**Further working seen**	**Action**
1.	$8\sqrt{2}$	$5.65685...$ *(incorrect decimal value)*	Award the final ***A1*** *(ignore the further working)*
2.	$\frac{1}{4}\sin 4x$	$\sin x$	Do not award the final ***A1***
3.	$\log a - \log b$	$\log(a-b)$	Do not award the final ***A1***

3 Implied marks

*Implied marks appear in **brackets e.g. (M1)**, and can only be awarded if **correct** work is seen or if implied in subsequent working.*

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

4 Follow through marks (only applied after an error is made)

*Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the answer (i.e. there is no working expected), then **FT** marks should be awarded if appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word "their" in a description, to indicate that candidates may be using an incorrect value.
- Exceptions to this rule will be explicitly noted on the markscheme.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.

5 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). Apply a **MR** penalty of 1 mark to that question*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- The ***MR*** penalty can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme

- Alternative methods for complete questions are indicated by ***METHOD 1***, ***METHOD 2***, *etc*.
- Alternative solutions for part-questions are indicated by ***EITHER*** . . . ***OR***.

7 Alternative forms

Unless the question specifies otherwise, ***accept*** *equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

8 Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. There are two types of accuracy errors, and the final answer mark should not be awarded if these errors occur.

- **Rounding errors**: only applies to final answers not to intermediate steps.
- **Level of accuracy**: when this is not specified in the question the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures*.

9 Calculators

A GDC is required for this examination, but calculators with symbolic manipulation features/ CAS functionality are not allowed.

Calculator notation
The subject guide says:

Students must always use correct mathematical notation, not calculator notation.

Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

1. (a) (i) $\mathrm{N} = 24$

$\mathrm{I\%} = 14$

$\mathrm{PV} = -14000$

$\mathrm{FV} = 0$

$\mathrm{P/Y} = 4$

$\mathrm{C/Y} = 4$ ***(M1)(A1)***

Note: Award ***M1*** for an attempt to use a financial app in their technology, award ***A1*** for all entries correct. Accept $\mathrm{PV} = 14000$.

$(€)871.82$ ***A1***

(ii) $4 \times 6 \times 871.82$ ***(M1)***

$(€)20923.68$ ***A1***

(iii) $20923.68 - 14000$ ***(M1)***

$(€)6923.68$ ***A1***

[7 marks]

(b) (i) $0.9 \times 14000\ (= 14000 - 0.10 \times 14000)$ ***M1***

$(€)12600.00$ ***A1***

(ii) $\mathrm{N} = 72$

$\mathrm{PV} = 12600$

$\mathrm{PMT} = -250$

$\mathrm{FV} = 0$

$\mathrm{P/Y} = 12$

$\mathrm{C/Y} = 12$ ***(M1)(A1)***

Note: Award ***M1*** for an attempt to use a financial app in their technology, award ***A1*** for all entries correct. Accept $\mathrm{PV} = -12600$ provided $\mathrm{PMT} = 250$.

$12.56(\%)$ ***A1***

[5 marks]

continued…

Question 1 continued

(c) ***EITHER***

Bryan should choose Option A ***A1***
no deposit is required ***R1***

Note: Award ***R1*** for stating that no deposit is required. Award ***A1*** for the correct choice from that fact. Do not award ***R0A1.***

OR

Bryan should choose Option B ***A1***
cost of Option A $(6923.69) >$ cost of Option B $(72 \times 250 - 12600 = 5400)$ ***R1***

Note: Award ***R1*** for a correct comparison of costs. Award ***A1*** for the correct choice from that comparison. Do not award ***R0A1***.

[2 marks]

(d) $14000\left(1-\frac{25}{100}\right)^{6}$ ***(M1)(A1)***

Note: Award ***M1*** for substitution into compound interest formula. Award ***A1*** for correct substitutions.

$= 2491.70$ (USD) ***A1***

OR

$\text{N} = 6$

$\text{I\%} = -25$

$\text{PV} = \pm 14000$

$\text{P/Y} = 1$

$\text{C/Y} = 1$ ***(A1)(M1)***

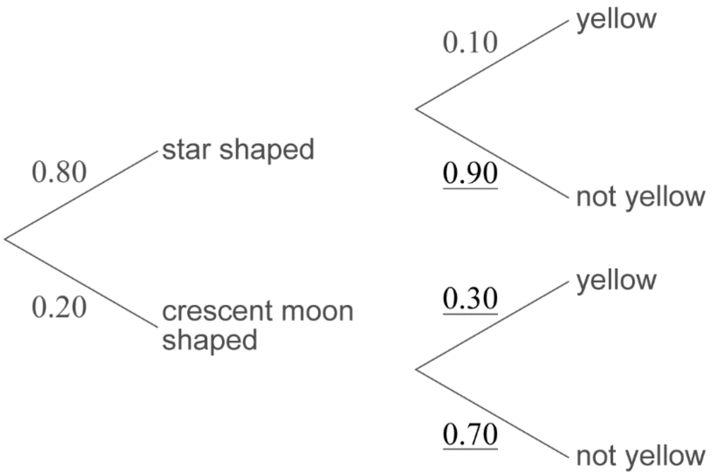
Note: Award ***A1*** for $\text{PV} = \pm 14000$, ***M1*** for other entries correct.

2491.70 (USD) ***A1***

[3 marks]

Total [17 marks]

2. (a)



A1A1

> **Note:** Award ***A1*** for each correct pair of branches. Accept decimal or percentage responses as equivalent forms on branches.

[2 marks]

(b) (i) $\mathrm{P}(Y) = 0.8 \times 0.1 + 0.2 \times 0.3$ ***M1***

$= 0.14$ ***A1***

(ii) $\mathrm{P}(\mathrm{Star} \mid Y) = \dfrac{0.8 \times 0.1}{0.14}$ ***M1***

$= 0.571 \left(\dfrac{4}{7}, 0.571428\ldots \right)$ ***A1***

[4 marks]

(c) the colours of the sweets are distributed according to manufacturer specifications ***A1***

[1 mark]

(d)

Colour	Brown	Red	Green	Orange	Yellow	Purple
Expected Frequency	12	20	16	16	8	8

A2

> **Note:** Award ***A2*** for all 6 correct expected values, ***A1*** for 4 or 5 correct values, ***A0*** otherwise.

[2 marks]

(e) 5 ***A1***

[1 mark]

(f) 0.469 (0.4688117…) ***A2***

[2 marks]

continued…

Question 2 continued

(g) since $0.469 > 0.05$ ***R1***

fail to reject the null hypothesis. There is insufficient evidence to reject the manufacturer's specifications ***A1***

Note: Award ***R1*** for a correct comparison of their correct p-value to the test level, award ***A1*** for the correct result from that comparison. Do not award ***R0A1***.

[2 marks]

Total [14 marks]

3. (a) (i) 0.909 $(0.909181\ldots)$ ***A2***

(ii) (very) strong and positive ***A1A1***

Note: Award ***A1*** for (very) strong ***A1*** for positive.

[4 marks]

(b) $y = 1.14x + 0.578$ $(y = 1.14033\ldots\ x + 0.578183\ldots)$ ***A1A1***

Note: Award ***A1*** for $1.14x$, ***A1*** for 0.578. Award a maximum of ***A1A0*** if the answer is not an equation in the form $y = mx + c$.

[2 marks]

(c) (i) $1.14 \times 10 + 0.578$ ***M1***

12.0 $(11.9814\ldots)$ ***A1***

(ii) no the estimate is not reliable ***A1***

outside the known data range ***R1***

OR

a score greater than 10 is not possible ***R1***

Note: Do not award ***A1R0***.

[4 marks]

(d)

Competitors	A	B	C	D	E	F	G	H
Stan's rank	7	8	6	4	2	4	1	4
Minsun's rank	7	8	6	4.5	3	2	1	4.5

A1A1

Note: Award ***A1*** for correct ranks for Stan. Award ***A1*** for correct ranks for Minsun.

[2 marks]

(e) (i) 0.933 $(0.932673\ldots)$ ***A2***

(ii) Stan and Minsun strongly agree on the ranking of competitors. ***A1A1***

Note: Award ***A1*** for "strongly agree", ***A1*** for reference to a rank order.

[4 marks]

(f) decreasing the score to 9.1, does not change the rank of competitor G ***A1***

[1 mark]

Total [17 marks]

4. (a) $2(8\times4+3\times4+3\times8)$ ***M1***

$=136\ (\text{cm}^2)$ ***A1***

[2 marks]

(b) $\sqrt{8^2+4^2+3^2}$ ***M1***

$(\text{AG} =)\ 9.43\ (\text{cm})\ (9.4339\ldots,\ \sqrt{89})$ ***A1***

[2 marks]

(c) $-2x+220=0$ ***M1***

$x=110$ ***A1***

$110\,000$ (boxes) ***A1***

[3 marks]

(d) $P(x)=\int -2x+220\,\mathrm{d}x$ ***M1***

Note: Award ***M1*** for evidence of integration.

$P(x)=-x^2+220x+c$ ***A1A1***

Note: Award ***A1*** for either $-x^2$ or $220x$ award ***A1*** for both correct terms and constant of integration.

$1700=-(20)^2+220(20)+c$ ***M1***

$c=-2300$

$P(x)=-x^2+220x-2300$ ***A1***

[5 marks]

(e) $-x^2+220x-2300=0$ ***M1***

$x=11.005$ ***A1***

$11\,006$ (boxes) ***A1***

Note: Award ***M1*** for their $P(x)=0$, award ***A1*** for their correct solution to x. Award the final ***A1*** for expressing their solution to the minimum number of boxes. Do not accept $11\,005$, the nearest integer, nor $11\,000$, the answer expressed to 3 significant figures, as these will not satisfy the demand of the question.

[3 marks]

Total [15 marks]

5. (a) (i) $p(10)^2 + q(10) = 60$ ***M1***

$10p + q = 6 \ (100p + 10q = 60)$ ***A1***

(ii) $p = 1,\ q = -4$ ***A1A1***

Note: If p and q are both incorrect then award ***M1A0*** for an attempt to solve simultaneous equations.

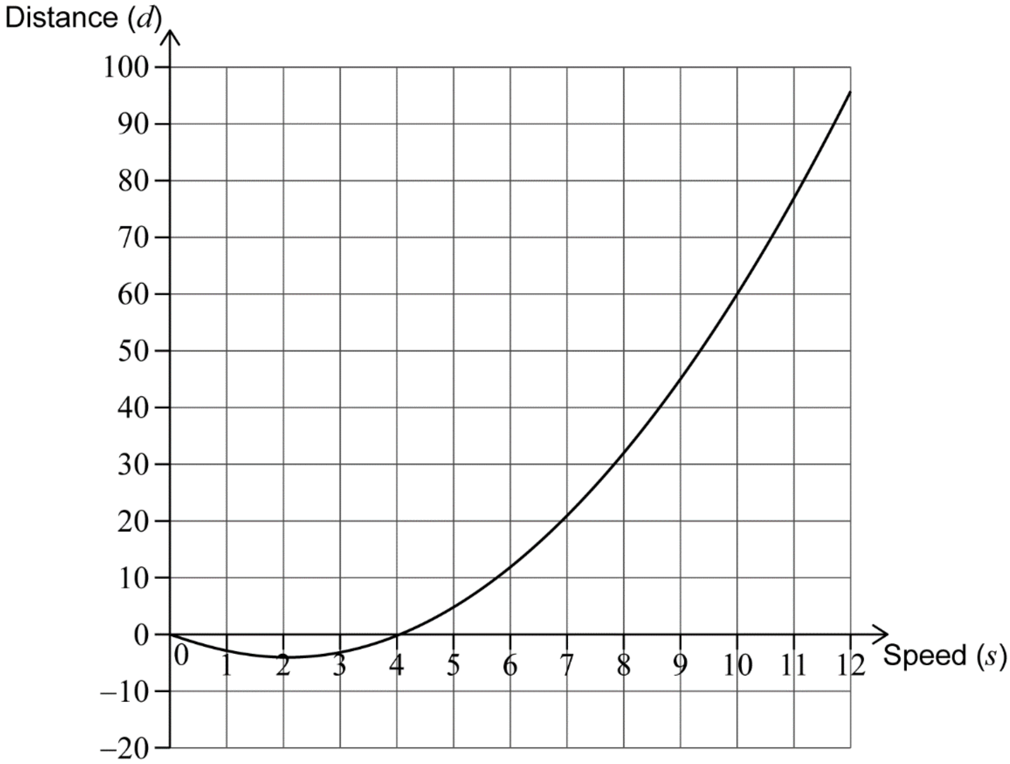
[4 marks]

(b) $(2, -4)$ ***A1A1***

Note: Award ***A1*** for each correct coordinate.
Award ***A0A1*** if parentheses are missing.

[2 marks]

(c)



A3

Note: Award ***A1*** for smooth quadratic curve on labelled axes and within correct window. Award ***A1*** for the curve passing through $(0, 0)$ and $(10, 60)$. Award ***A1*** for the curve passing through their vertex. Follow through from part (b).

[3 marks]

(d) the graph indicates there are negative stopping distances (for low speeds) ***R1***

Note: Award ***R1*** for identifying that a feature of their graph results in negative stopping distances (vertex, range of stopping distances…).

[1 mark]

continued…

Question 5 continued

(e) $0.95\times 20^2 - 3.92\times 20$ ***(M1)***

$= 302\,(\text{m})\ (301.6\ldots)$ ***A1***

[2 marks]

(f) $\left|\dfrac{301.6-320}{320}\right|\times 100$ ***M1***

$= 5.75\,(\%)$ ***A1***

[2 marks]

(g) $330 = 1.6\times s + 0.95\times s^2 - 3.92\times s$ ***M1A1***

Note: Award ***M1*** for an attempt to find an expression including stopping distance (model B) and reaction distance, equated to 330.
Award ***A1*** for a completely correct equation.

$19.9\,(\text{m s}^{-1})\ (19.8988\ldots)$ ***A1***

[3 marks]

Total [17 marks]